Mathematics 13

UNIT 5
Relations

Distance Learning

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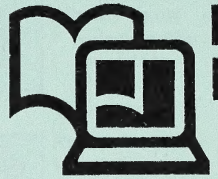
Welcome



Distance Learning

You have chosen an alternate form of learning that allows you to work at your own pace. You will be responsible for your own schedule, for disciplining yourself to study the units thoroughly, and for completing your units regularly. We wish you much success and enjoyment in your studies.

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General Information

This information explains the basic layout of each booklet.

- **What You Already Know** and **Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.
- As you begin each **Topic**, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.
- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.
- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.
- **Extensions** gives you the opportunity to take the topic one step further.
- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.
- The **Appendices** include the solutions to **Activities (Appendix A)** and any other charts, tables, etc. which may be referred to in the topics **(Appendix B, etc.)**.

Visual Cues

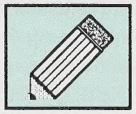
Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

An explanation of what they mean is written beside each visual cue.

What You Already Know
- reviewing what you already know

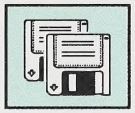
Key Idea
- flagging important ideas

Audiotape
- learning by listening to an audiotape



Review
- studying previous concepts

Another View
- exploring different perspectives



Computer Software
- learning by using computer software

Introduction
- introducing the unit

Solutions
- correcting the activities

Videotape
- learning by viewing a videotape

What Lies Ahead
- previewing the unit

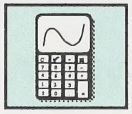
Extra Help
- providing additional study

Print Pathway
- choosing a print alternative

Exploring the Topic
- actively learning new concepts

Extensions
- going on with the topic

Calculator
- using your calculator



Graphing Calculator
- using your graphing calculator

What You Have Learned
- summarizing what you have learned

Mathematics 13

Course Overview

Mathematics 13 contains 6 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 Number Systems	10%
Unit 2 Polynomials and Factoring	28%
Unit 3 Geometry	18%
Unit 4 Coordinate Geometry and Graphing	18%
Unit 5 Relations	**12%**
Unit 6 Statistics	14%
	100%

Unit Assessment

After completing the unit, you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal, your teacher will determine what this assessment will be. It may be

Unit Assignment - 50%
Supervised Unit Test - 50%

Introduction to Relations

This unit covers topics dealing with relations. Each topic contains explanations, examples, and activities to assist you in understanding relations. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help**. If you would like to extend your knowledge of the topic, there is a section called **Extensions**.

You can evaluate your understanding of each topic by working through the activities. Answers are found in the solutions in the **Appendix**. In several cases there is more than one way to do the question.

Unit 5 Relations

Contents at a Glance

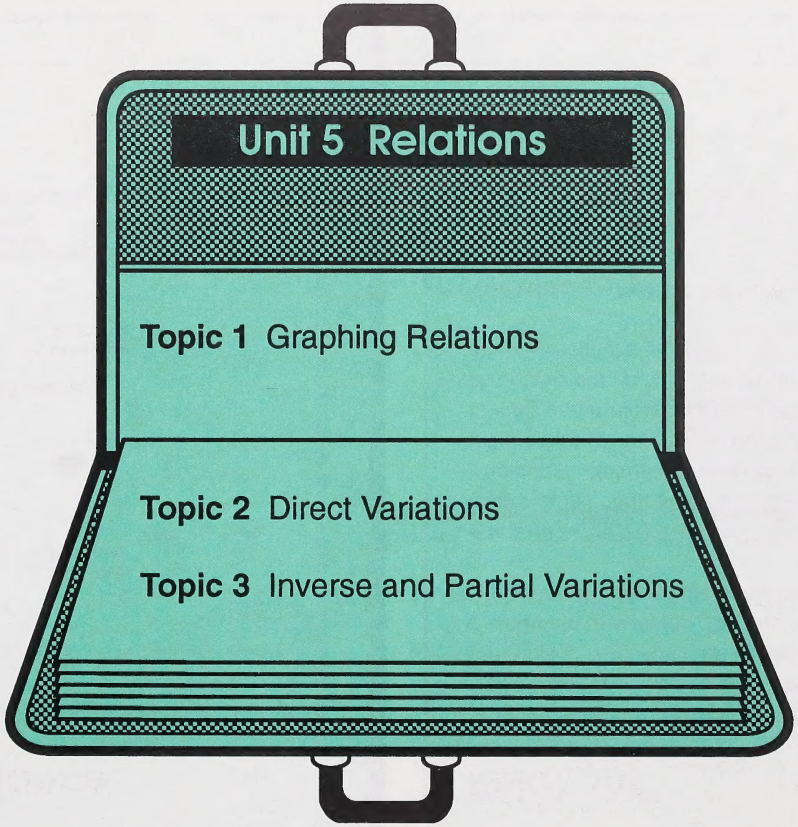
Value		
	Relations	3
	What You Already Know	5
	Review	6
11%	**Topic 1:** Graphing Relations	8
	• Introduction • Extra Help • What Lies Ahead • Extensions • Exploring Topic 1	
26%	**Topic 2:** Direct Variations	44
	• Introduction • Extra Help • What Lies Ahead • Extensions • Exploring Topic 2	
63%	**Topic 3:** Inverse and Partial Variations	64
	• Introduction • Extra Help • What Lies Ahead • Extensions • Exploring Topic 3	
	Unit Summary	93
	• What You Have Learned • Unit Assignment	
	Appendix	94

Relations

Mathematics is often used to describe relationships in the world around you. For example, as a candle burns, its height diminishes. The weathering action of wind and rain can reduce a mountain to a plain. The rate of the erosion can help scientists determine the age of the continent where that mountain was.

It was through the use of relations that the temperature of the centre of the earth and the temperature of the sun were calculated. Scientists could also tell that deep space is very cold, although no one has been there. Once you understand inverse and direct relations, a whole world of information can be at your command. The world of research and development moves along this path.

You will get a feel for this type of reasoning in this unit.

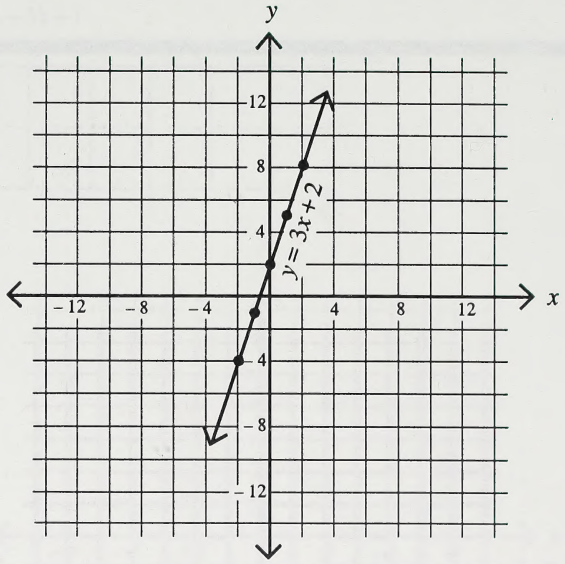

Unit 5 Relations
Topic 1 Graphing Relations
Topic 2 Direct Variations
Topic 3 Inverse and Partial Variations

What You Already Know

Recall that the graph of any equation can be found by completing a table of values. The ordered pairs generated are used to plot the graph.

For example, what does the graph of $y = 3x + 2$ look like? The solution is as follows:

x	-2	-1	0	1	2
y	-4	-1	2	5	8



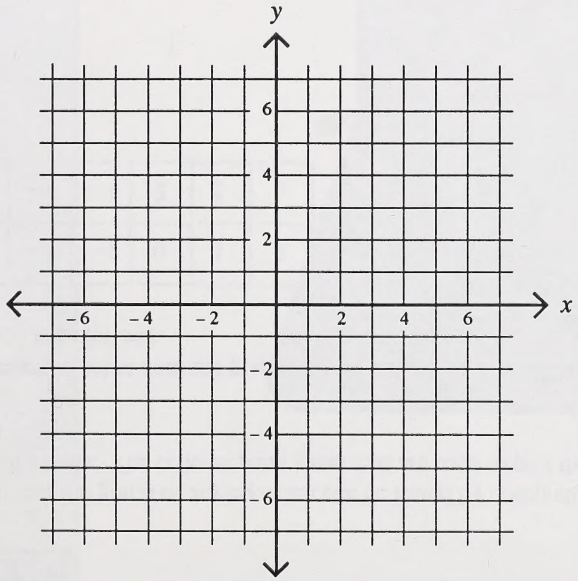
Try the following questions to check your understanding.

Review

For each of the following equations, complete a table of values and draw the graph using the resulting ordered pairs.

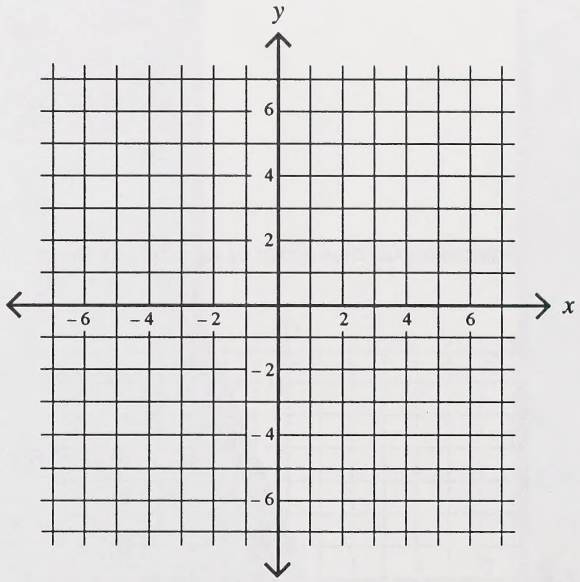
1. $y = 2x$

x					
y					



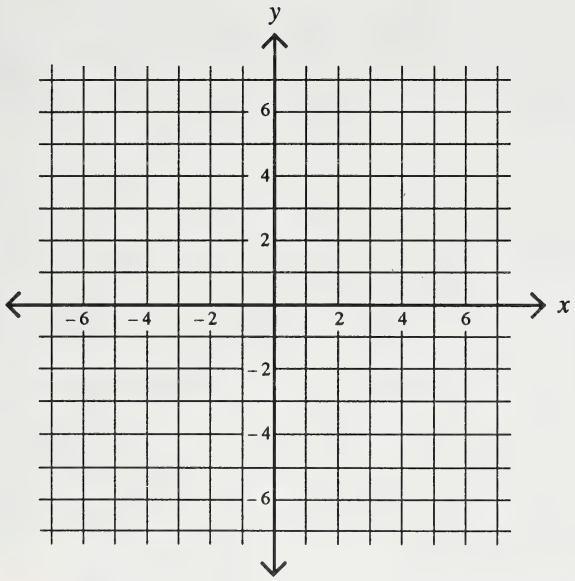
2. $y = -2x$

x					
y					



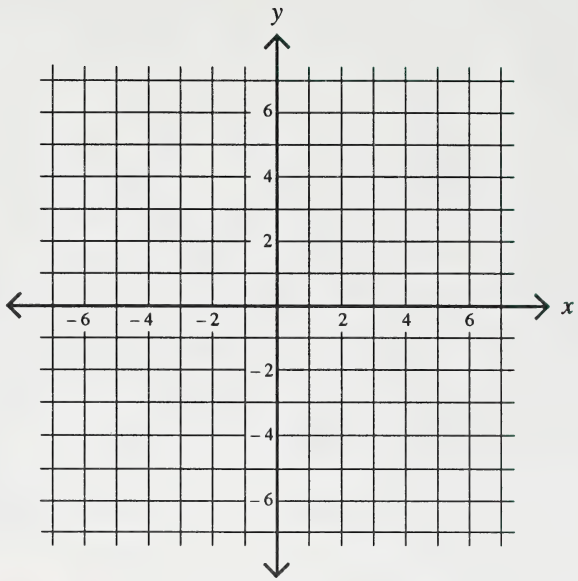
3. $y = 3x - 1$

x					
y					



4. $y = -3x + 1$

x					
y					



Now go to the **Review** solutions in the **Appendix**.

Topic 1 Graphing Relations

Introduction

People progress by understanding the world around them. The more people deal with and understand relationships, the better they are able to function in the world.

Consider the following:

- If you drive far enough, you will run out of gas.
- The harder you hit a ball, the farther it will go.
- The more sugar you consume, the more cavities you are likely to have.
- The colder it gets, the more clothing you will need to wear.

This line of thinking will be developed in this topic.

What Lies Ahead

Throughout the topic you will learn to

1. sketch graphs that relate to physical phenomena, and interpret graphs that deal with real-life variables
2. interpolate and extrapolate data from graphs and tables

Now that you know what to expect, turn the page to begin your study of graphing relations.

Exploring Topic 1

Activity 1

Sketch graphs that relate to physical phenomena, and interpret graphs that deal with real-life variables.

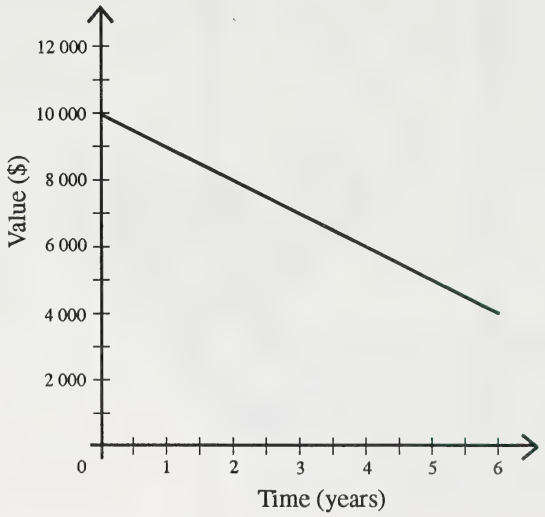
Ideas can be expressed in various ways. You can express an idea orally or visually or with the written word.

Information can be communicated in concise and compact form. A graph is one way of conveying or relating certain forms of information. It can also summarize an idea quite easily.

The following examples show how one value often depends on another.

Example 1

In the following graph, what happens to the value of the car as time increases?

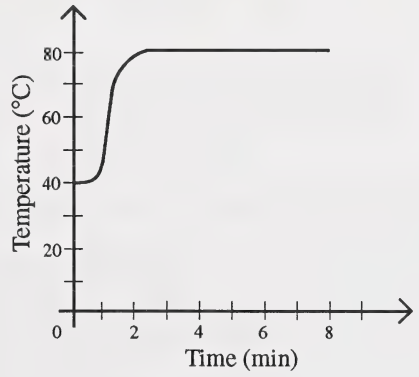


Solution:

As the age of the car increases, its value decreases. The value of the car depends on the age of the car.

Example 2

A hot-water tap is turned on and allowed to run. The following graph shows the change in the temperature of the running water over an eight-minute period. Study the graph and answer the following questions.



- What happens to the water temperature as the time increases to two minutes?

  Solution:

  As the time increases from zero to two minutes, the water temperature increases.

- What happens to the water temperature at any time between three and seven minutes?

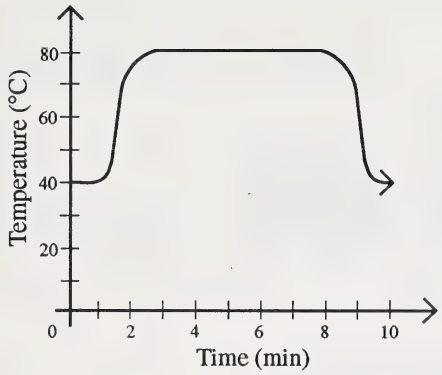
  Solution:

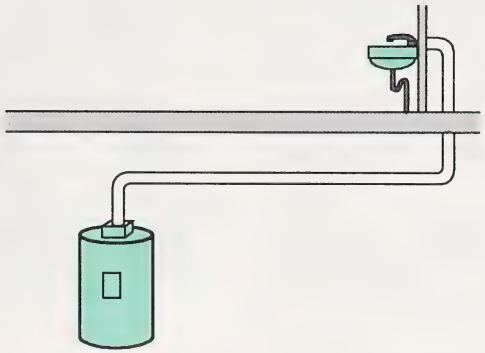
  For the time between three and seven minutes, the temperature of the tap water remains unchanged.

- What would be the effect on the graph if the tap was left on indefinitely?

Solution:

Once all of the hot water is gone, the temperature of the water would fall. The graph would look as follows.



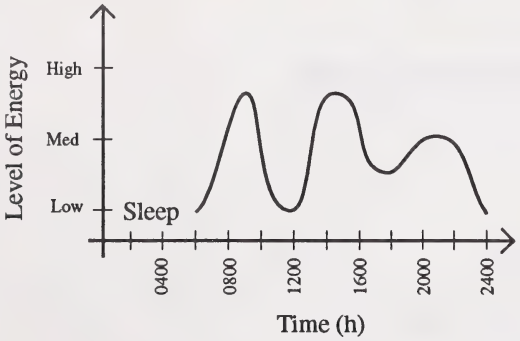


You could say that the temperature of the water coming out of the hot-water tap depends on the amount of time that the tap is left on.

Example 3

Renata has times of high and low energy levels. Study Renata's energy curve, and answer the following questions.

Renata's Energy Curve



- How is Renata's energy level affected by the time of day?

  Solution:

  According to Renata's energy curve, her energy level fluctuates with the time of day.

- How might you explain the changes in energy level between 1200 and 1400?

  Solution:

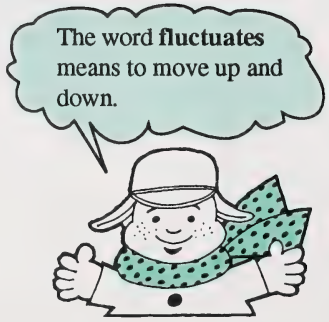
  Between 1200 and 1400, Renata's energy level rises. This could be a result of eating lunch.

- How might you explain the changes in energy level between 1400 and 1800?

  Solution:

  Renata's energy level falls because she probably gets tired and hungry between 1400 and 1800.

In Examples 1 to 3, each graph shows that one changing idea or **variable** is connected to another. The resulting pattern of change is called a **relation**. A relation is defined by a set of ordered pairs.



In many instances, phenomena found in nature can be related. Once the relation is found and identified, you can express the relation quite explicitly on a graph.

Consider the following:

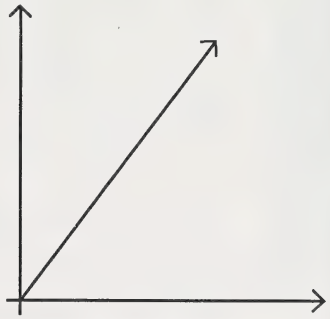
- People get stronger and more alert as they grow older. After reaching their prime, they begin to get weaker and less alert.
- The process of erosion reduces mountains to plains. Canyons grow wider with time.
- People tend to forget things with the passing of time.

Graphs reflect relations as **increasing**, **decreasing**, **fluctuating**, or **remaining constant**. In the examples that follow, some familiar situations and relations will be presented and graphed.

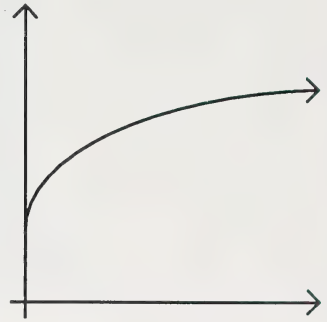
Example 4

Consider the following relations and find the graph that best portrays the situation. In each case, label the axes correctly.

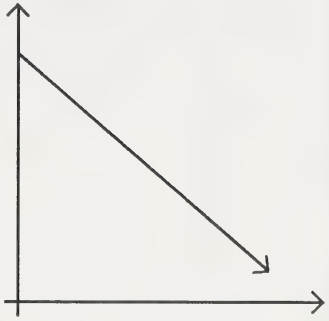
Graph *A*



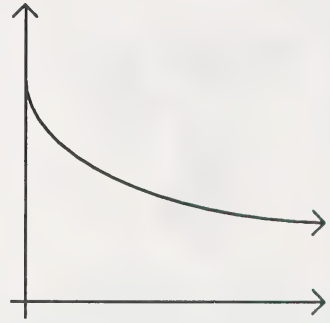
Graph *B*



Graph *C*



Graph *D*



- The value of a television set decreases over a long period of time.

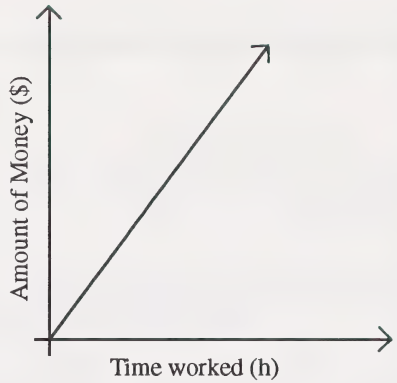
  Solution:

  The graph that best shows this is Graph *D*. The value of a television set will decrease with time. At first, there will be a rapid decrease in value; then, the decrease will be more gradual.

- The more hours you work, the more money you earn.

  Solution:

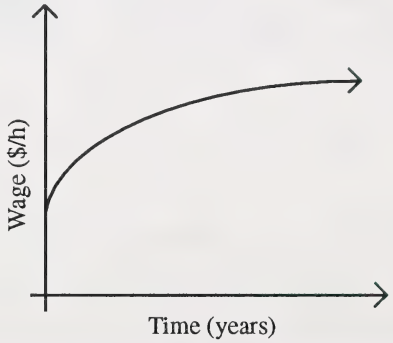
  The graph that best shows this is Graph *A*. The amount of money you earn will increase as you work more hours.

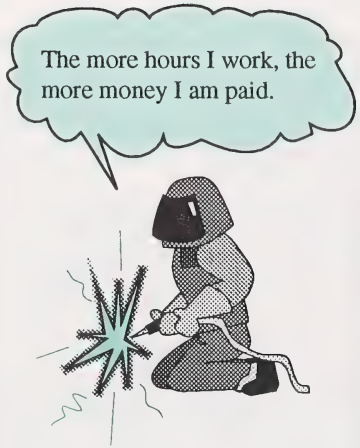


- The wage of an average person increases with years of experience on the job.

  Solution:

  The graph that best shows this is Graph *B*. The wage increases rapidly in the first year on the job. Increases will then slow down and may eventually cease.

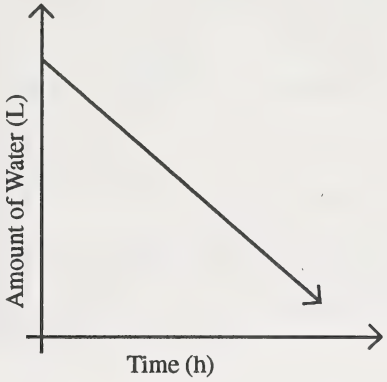




- The longer a holding tank leaks, the less water it will contain.

Solution:

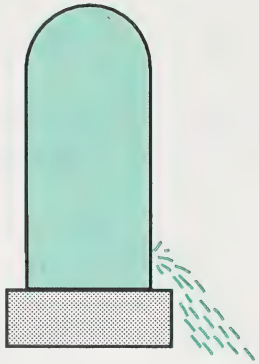
The graph that best shows this is Graph *C*. The amount of water in a leaky holding tank will decrease uniformly as time goes by.



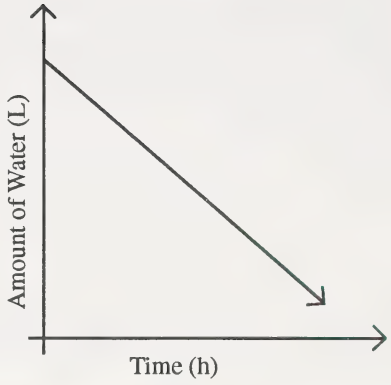
You should note that the time axis is generally the horizontal one. Later in this topic you will learn why certain variables are generally placed along the *y*-axis, while others are placed along the *x*-axis.

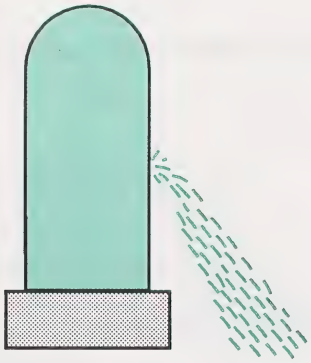
In the last few examples, you saw graphical representation of relations of various types. In the next few examples, you will be given the relations and then asked to construct the graphical representation.

Would the graph depicting a holding tank that leaks from the bottom be the same as the graph depicting a holding tank that leaks from a point part way up the tank? The answer to this question can be found by studying the following two diagrams and graphs. Examine each one carefully and notice the difference in the graphs.

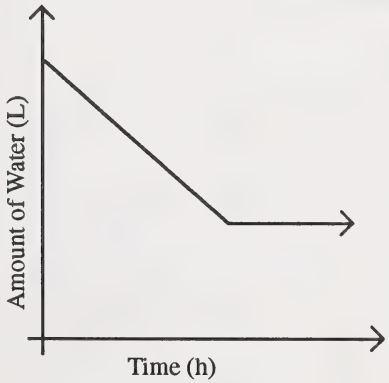


Leak From Bottom





Leak From Side

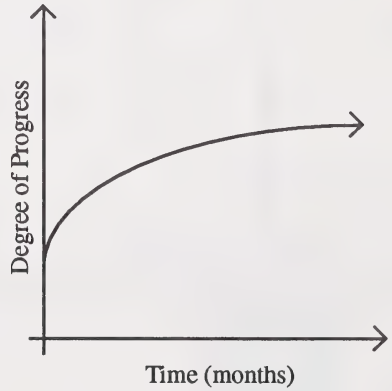


Example 5

Make a rough sketch of the graph depicting the following relations.

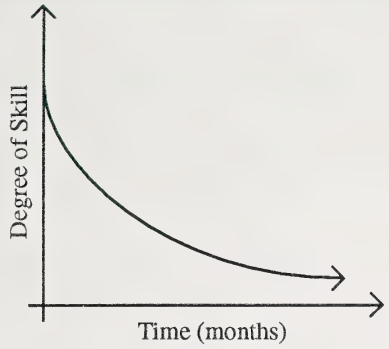
- A person wants to learn to ride a bicycle. The person's skills will develop rapidly at first and then the degree of development will level off.

  Solution:



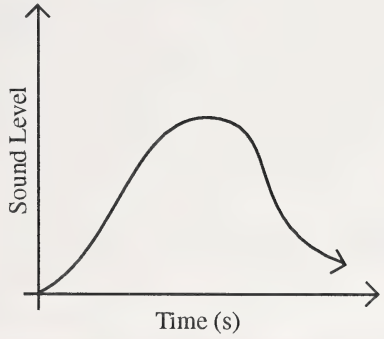
- Without practice, the skills of a person who once knew how to type will decrease rapidly at first and then more slowly after a period of time.

Solution:



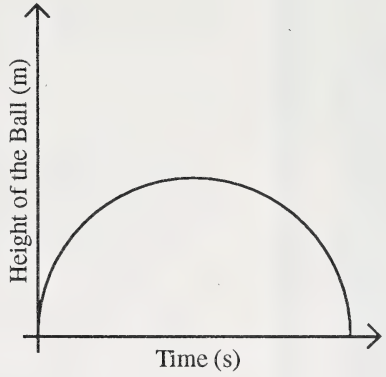
- The sound of a train is quieter when it is far away than when it is nearby. As the train approaches, the sound level increases and reaches a peak; then the sound level decreases as the train continues along its journey.

Solution:



- When a ball is thrown into the air, it rises and reaches its highest point and then falls back to earth.

Solution:



A relationship can be expressed in the form of a **table of values**. This table contains ordered pairs which can be plotted on a coordinate plane to give a clearer picture of the relation.

Consider the following example.

Example 6

The following table relates the distance in kilometres that a rocket travels in specific time intervals (up to 21 s in all).

Organize the table of values into ordered pairs and plot the points on a coordinate plane. Complete the graph by joining the points with a smooth curve.

Time (s)	0	3	6	9	13	16	19	20	21
Distance (km)	0	2	4	8	14	22	34	40	48

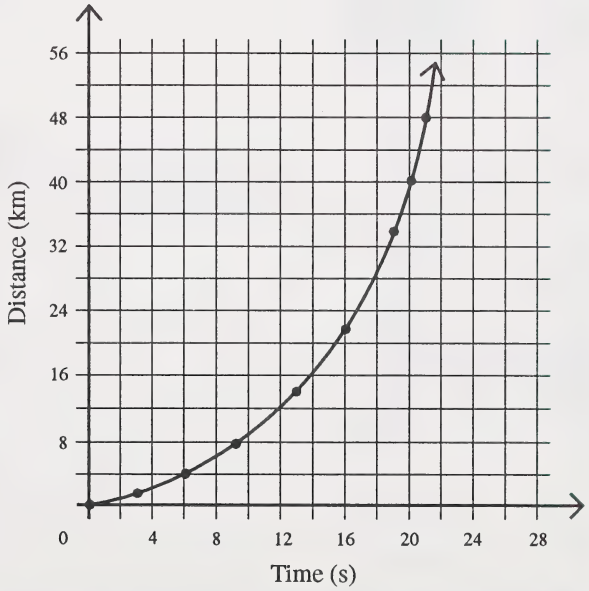
Solution:

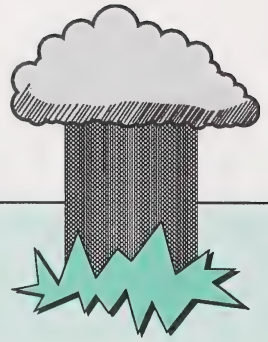
The ordered pairs are as follows:

$(0, 0)$, $(3, 2)$, $(6, 4)$,

$(9, 8)$, $(13, 14)$, $(16, 22)$,

$(19, 34)$, $(20, 40)$, $(21, 48)$





Notice the difference in labelling the horizontal and vertical axes. The vertical axis increases by four while the horizontal axis increases by two. It is not necessary to number the intervals the same way on both axes.

You have graphed relations by locating a variable on each axis. Now you will also learn a rule for labelling the axes.

How is the size of your shoe and the size of your foot related? The size of your shoe depends on the size of your foot. The size of your shoe is a dependent variable. The dependent variable is placed on the vertical axis.

The other variable is called the independent variable and is placed along the horizontal axis.

The dependent variable is always located on the vertical axis of the graph.

The independent variable is always located on the horizontal axis of the graph.

You choose the values for the independent variable. The values for the dependent variable are calculated from the values of the independent variable. These can be used to generate a **table of values**. Tables of values are used to generate ordered pairs. Ordered pairs are used to construct a graph showing the relation.

Some relationships are known so well that a formula is used to make the statement about the relationship. Here is such an example.

Example 7

If the radius (r) of a circle is known, then the circumference (C) can be calculated using the formula $C = 2\pi r$. Prepare a table of values and draw the graph showing the relationship between C and r.

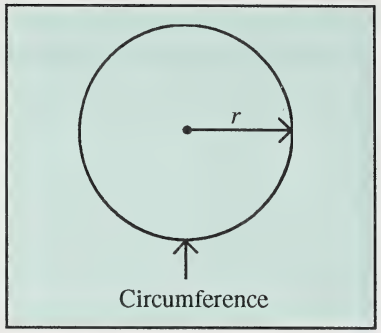
Solution:

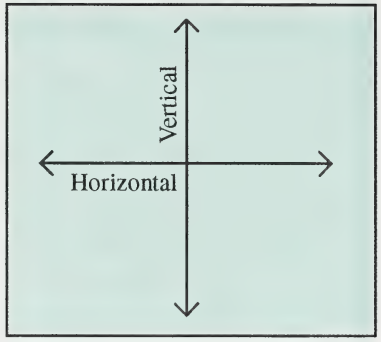
Step 1: Choose some convenient values for r. Any positive values could be chosen here. Consider $r = 0$ cm, $r = 2$ cm, $r = 5$ cm, and $r = 8$ cm.

Step 2: Use $C = 2\pi r$ to calculate C for each r. Use 3.14 as the value for π. The complete calculations are as follows:

When $r = 0$, $C = 2\pi r$
$= 2 \times 3.14 \times 0$
$= 0$ cm

When $r = 1$, $C = 2\pi r$
$= 2 \times 3.14 \times 1$
$= 6.28$ cm





When $r = 2$, $C = 2\pi r$

$= 2 \times 3.14 \times 2$

$= 12.56$ cm

When $r = 5$, $C = 2\pi r$

$= 2 \times 3.14 \times 5$

$= 31.40$ cm

When $r = 8$, $C = 2\pi r$

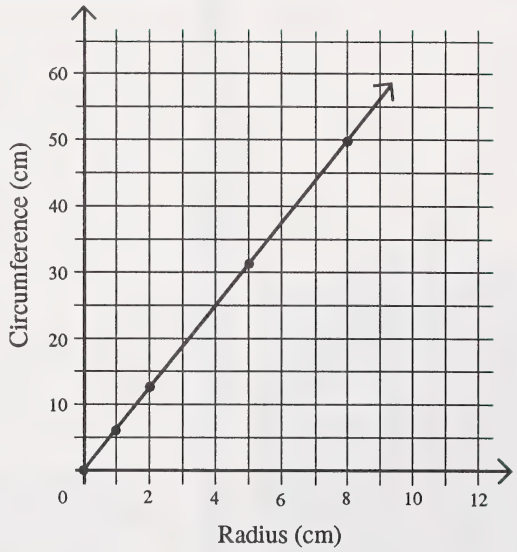
$= 2 \times 3.14 \times 8$

$= 50.24$ cm

r	0	1	2	5	8
C	0	6.28	12.56	31.40	50.24

Step 3: Consider the numbers in the table as ordered pairs, and graph $(0, 0)$, $(1, 6.28)$, $(2, 12.56)$, $(5, 31.40)$, and $(8, 50.24)$. You must find the appropriate positions for all the points.

Step 4: Join the points.



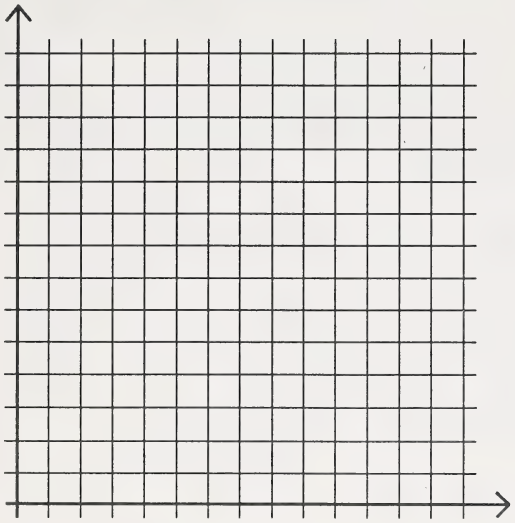
The fact that the points all lie on the same straight line indicates that the relationship between the circumference and the radius is continuous and direct.

The questions that follow will give you more experience with relations.

Do questions 1 to 3 and any other two questions.

1. The number of litres (L) of gasoline used by a car depends on the distance (d) it travels. Use the following data table to draw a graph of this relation. (Use the vertical axis for L.)

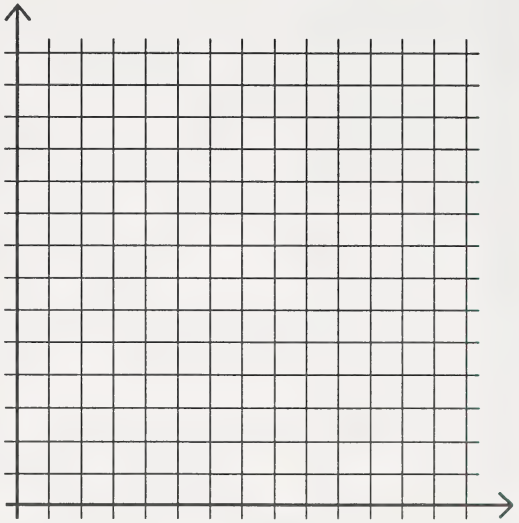
d	0	100	200	300	400	500
L	0	8	16	24	32	40



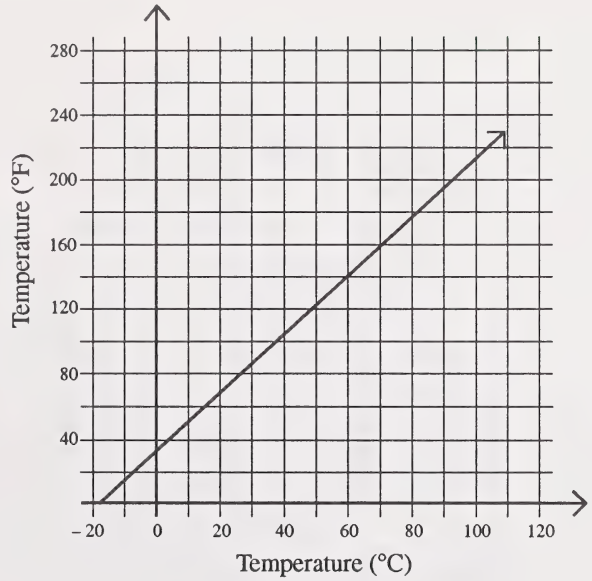
2. Marcus heats some water from room temperature (20°C) to boiling point (100°C). The water reaches boiling point after sixteen minutes. Draw a graph of the relation between the temperature of the water and the time. Using the following table. Explain your graph.

Temp	20	30	40	50	60	70	80	90	100	100	100
Time	0	2	4	6	8	10	12	14	16	18	20

Note that water does not get any hotter than 100°C at normal pressure.



3. Study the relationship between Celsius and Fahrenheit temperatures. Use the given graph to complete the table of values that follows.



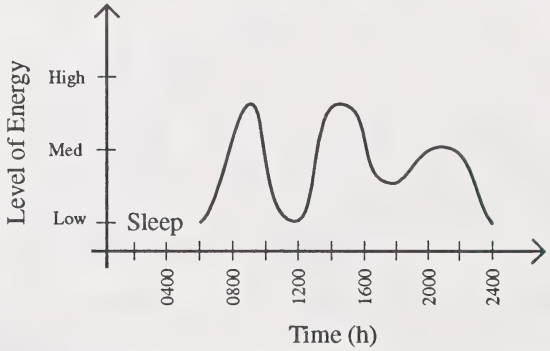
	CP	SP	FP	RT	BT	BP
°C	–40	–18			37	
°F	–40		32	70		212

Note that $(-40, -40)$ has **not** been plotted on the graph.

The abbreviations are explained as follows:

- CP = Cross Point where °C = °F
- SP = Solution Point (freezing point of salt and water)
- FP = Freezing Point for pure water
- RT = Room Temperature
- BT = Body Temperature
- BP = Boiling Point of pure water at sea level

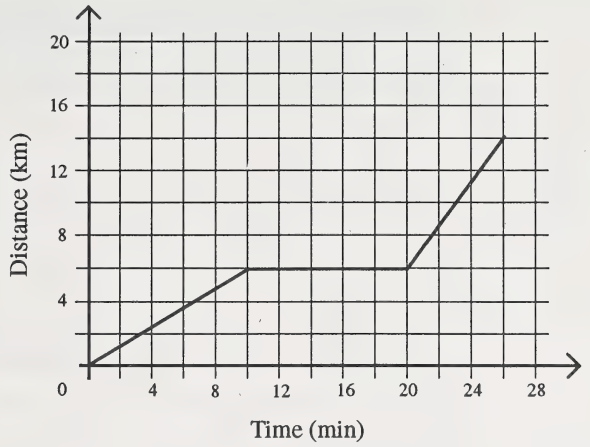
4. Renata's energy curve shows high and low energy levels at various times of the day.



a. Give some possible reasons as to why Renata's energy level should peak about 0800, 1400, and 2100.

b. At what times did Renata's energy level drop to a really low point?

5. Verica was driving to church. She stopped on the way to talk with a friend. The following graph gives information about Verica's travel to church.



a. How long did Verica stop to talk with her friend?

b. How far from Verica's home is the church?

c. How long would the trip have taken if Verica had gone straight to church?

6. Distance (d) travelled in kilometres is related to time (t) in minutes. This statement can be expressed in the form $d = 5t$.

a. What is the independent variable?

b. What is the dependent variable?

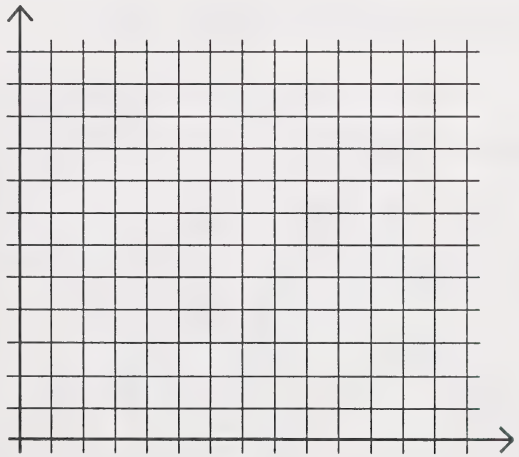
c. What is the distance travelled if the time is three minutes?

7. a. Complete the table of values for $i = 5t$, where i is interest and t is time.

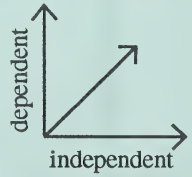
t	0	1		3	4		6		8	9
i			10			25		35		

b. State which is the independent variable and which is the dependent variable.

c. Draw a graph using the values from the table as ordered pairs.



The dependent variable is the one that changes because of changes in the independent variable.



The dependent variable is always located on the vertical axis. The independent variable is always located on the horizontal axis.

For solutions to **Activity 1**, turn to the **Appendix, Topic 1**.

Activity 2

Interpolate and extrapolate data from graphs and tables.

In the last activity it was demonstrated that a relationship between two quantities or variables can be shown using a graph. In the next few examples you will find some values for variables that are **not plotted** on the graph.

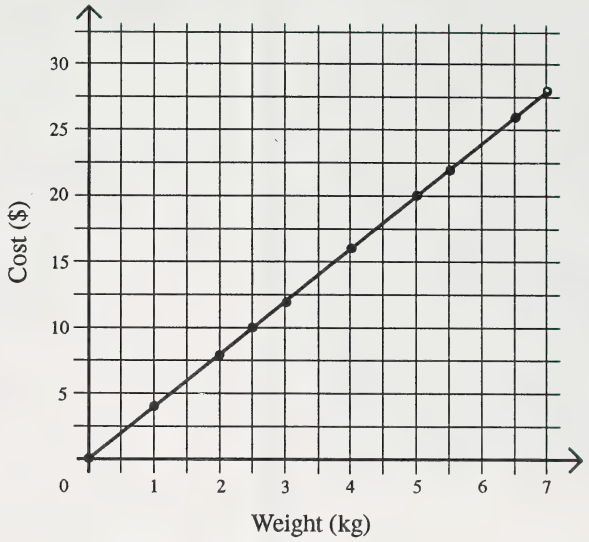
Example 8

The following table relates the weight of canola to its cost.

Weight (kg)	0	1	2	2.5	3	4	5	5.5	6.5	7
Cost ($)	0	4	8	10	12	16	20	22	26	28

- Make a graph using the ordered pairs from the table of values.

Solution:



- Use the graph to find the cost of 6 kg of canola.

Solution:

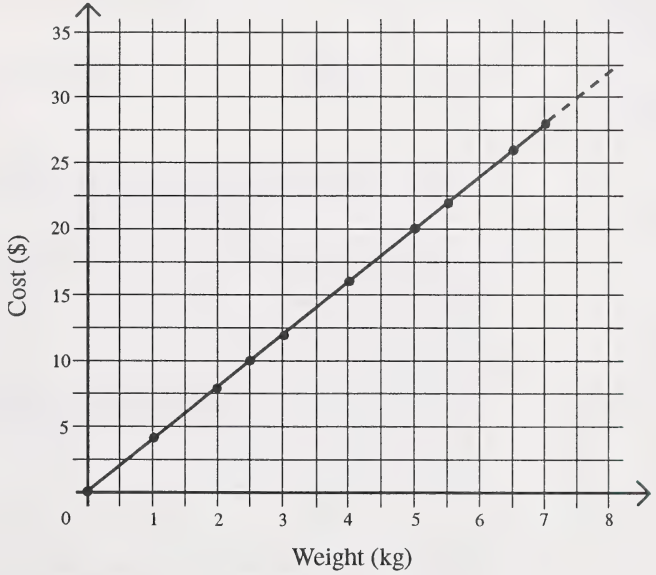
The cost of 6 kg of canola is not in the table of values and is not plotted on the graph. You need to find information between two given values from the table of values. This process is called **interpolation**.

To find the cost of 6 kg of canola, find 6 on the horizontal axis. Move vertically until you cross the line of the graph. At this point go back horizontally until you get to the vertical axis. From the graph, you can see that the cost of 6 kg of canola is about $24.

- Use the graph to find the cost of 7.4 kg of canola.

Solution:

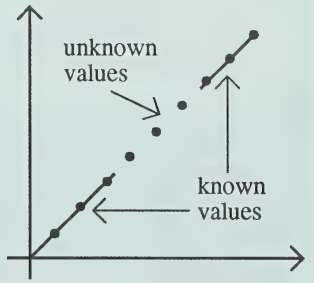
If the line of the graph is not long enough to give the reading of a larger quantity, you might have to extrapolate. If you wanted to buy 7.4 kg of canola, you cannot read the cost from the graph because the graph only goes to 7 kg. You must lengthen the line of the graph as shown.



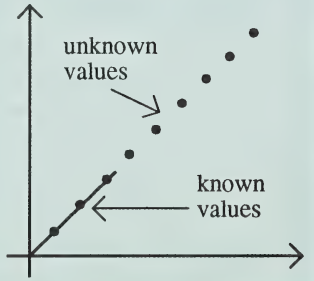
From the lengthened graph, you can find the cost of 7.4 kg of canola. Find 7.4 kg on the horizontal axis and move up until you cross the line of the graph. At this point, go back horizontally until you get to the vertical axis. From the graph, you can see that 7.4 kg of canola will cost about $30.

The process of finding information beyond the given data is called extrapolation.

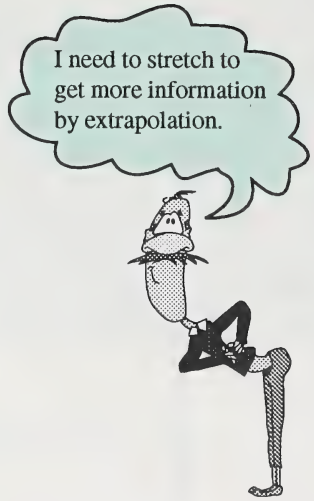
Interpolation is the process of finding new information between two given data or readings.



Extrapolation is the process of finding new information beyond the given data.

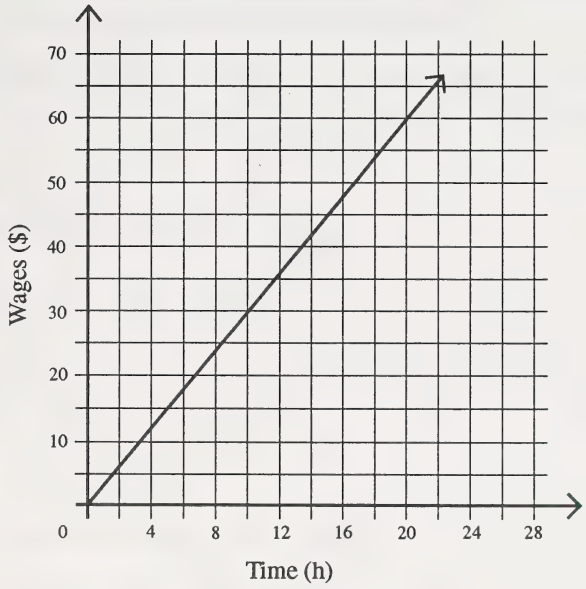


Sometimes extending the graph may be too time-consuming and the readings may be inaccurate. You may want to use a proportion to calculate the desired information. The next example shows how these calculations are done. You should notice that the calculated results are more accurate than the readings done through extrapolation.



Example 9

The following graph shows a relation between the wages earned in a factory in 1960 and the number of hours worked.



- How much would be earned in 35 h?

  Solution:

  According to the graph, you would earn \$30 for 10 h of work. Therefore, you would earn three dollars for one hour. In 35 h you would earn $35 \times \$3 = \$105$.

  You could use a proportion to find the answer.

$$\frac{\$30}{10\text{ h}} = \frac{x}{35\text{ h}}$$
$$10x = 30 \times 35$$
$$x = 1050 \div 10$$
$$= \$105$$

  In 35 h you would earn \$105.

- How long would it take to earn \$150?

  Solution:

  From the graph, you would earn \$30 for 10 h of work. Therefore, you would earn three dollars for one hour. To earn \$150, you would have to work $\$150 \div \$3/\text{h} = 50\text{ h}$.

  You could set up a proportion to find the answer.

$$\frac{\$30}{10\text{ h}} = \frac{\$150}{x\text{ h}}$$
$$30x = 10 \times 150$$
$$x = 1500 \div 30$$
$$= 50$$

  You would have to work 50 h to earn \$150.

The relationship between two variables is sometimes linear but discontinuous. In other words, the graph of the relation defines a path that appears to be along a straight line, but instead the graph is a set of dots along the path of a line. This happens when you have a number of things that must be kept whole. For example, shirts, cows, or bulbs must be whole. You cannot graph half a shirt, half a cow, or half a bulb.

Try an example of this kind.

Example 10

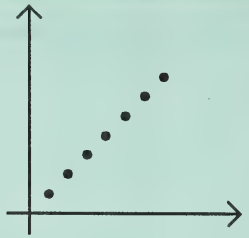
A 4-H club organized a fund-raising event. It bought twelve boxes of chocolates at $8 per box. The club made a $3 profit per box by selling the boxes at $11 each.

- Write the ordered pairs comparing the sale of the twelve boxes of chocolates with the amount of profit made.

Solution:

First make a table of values.

Number of Boxes of Chocolates	0	1	2	3	4	5	6	7	8	9	10	11	12
Total Profit	0	3	6	9	12	15	18	21	24	27	30	33	36

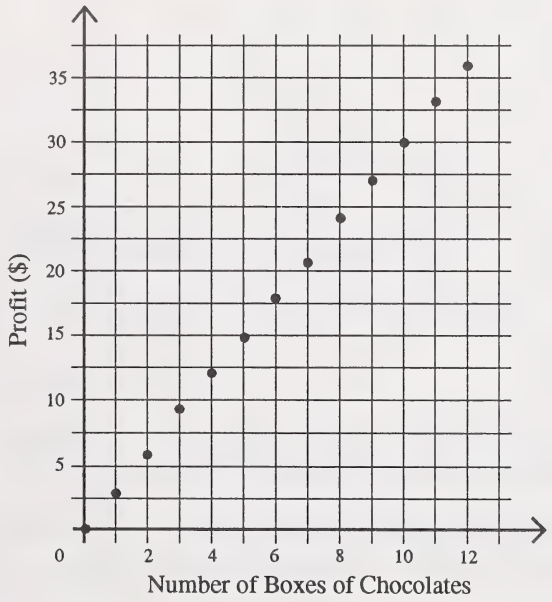


This shows a linear relationship which is discontinuous.

The ordered pairs are (0, 0), (1, 3), (2, 6), (3, 9), (4, 12), (5, 15), (6, 18), (7, 21), (8, 24), (9, 27), (10, 30), (11, 33), and (12, 36).

- Graph the ordered pairs.

Solution:



- Why would it be wrong to draw a line joining all the points?

Solution:

It would be wrong to join the points to form a straight line because this would suggest that you could buy part of a box of chocolates. In reality, chocolates are sold in whole boxes only. For example, joining the points between four and five boxes of chocolates would suggest that you could buy $4\frac{1}{2}$ boxes of chocolates. Therefore, these points must remain separate.

Even though a line should not be drawn through the points, it is still possible to extend the pattern found in the graph. In other words, extrapolation can be done to obtain information that is beyond the scope of this graph. For example, the profit from the sale of 27 boxes of chocolates would be $81.

There are times when very little is known about the relationship between two variables. When this is the case, you might have to perform an experiment, record the readings you obtain, and draw a graph. In the next example you will see how this is done.

Example 11

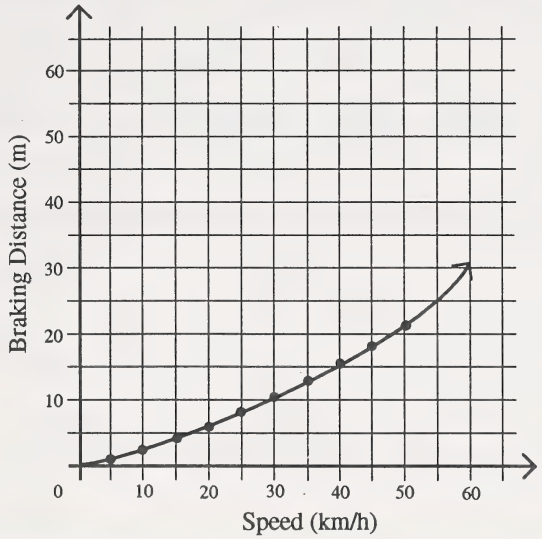
An experiment was conducted to see how quickly a snowmobile would come to a full stop after applying the brakes.

The following table shows the observations from the experiment.

Braking Distance (m)	1.3	2.7	4.2	6	7.9	10.1	12.6	15.3	18.4	22
Speed (km/h)	5	10	15	20	25	30	35	40	45	50

- Draw the graph of the curve from the data of the experiment.

Solution:



- If the snowmobile were travelling at 32 km/h, interpolate from the graph the distance required to come to a stop.

  Solution:

  By interpolation, it would take about 11 m for the snowmobile to come to a stop if it were travelling at 32 km/h.

- A rider discovers that it took 26 m for the snowmobile to come to a full stop. How fast was the rider going?

  Solution:

  If it took the rider 26 m to bring the snowmobile to a halt, the rider must have been travelling at about 55 km/h.

Now that you know how to read and understand graphs of relations, try the following questions.

Do at least four of the following questions.

1. The following table shows a relation between the amount of money earned in dollars and the amount of time worked in hours.

Time (h)	0	1	2	3	4	5	6	7	8	9	10	11	12
Earnings ($)	0			15		25			40			55	

   a. Complete the previous table.

b. Find the amount earned for the following hours worked.

i. 3 h ii. 4 h

iii. 11 h iv. 5.5 h

v. 10.5 h

c. Find how long it would take to earn the following amounts.

i. $35 ii. $21

iii. $30 iv. $105

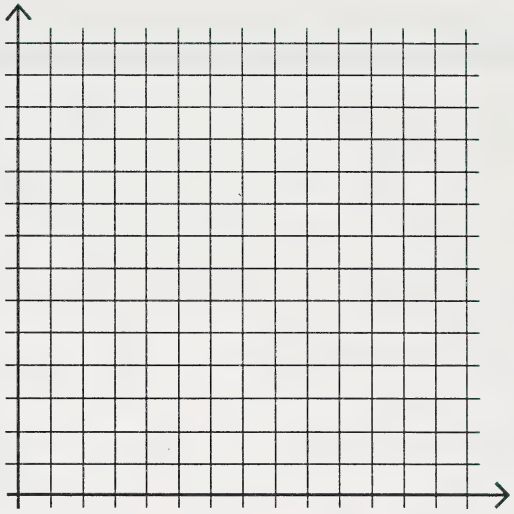
v. $43.50

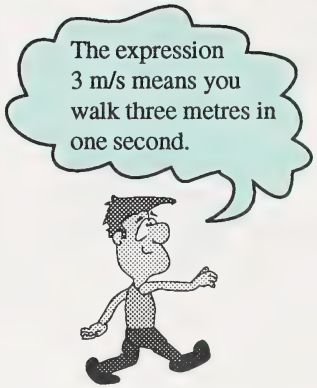
2. Donna walks at a rate of 3 m/s.

a. How far can she walk in 21 s?

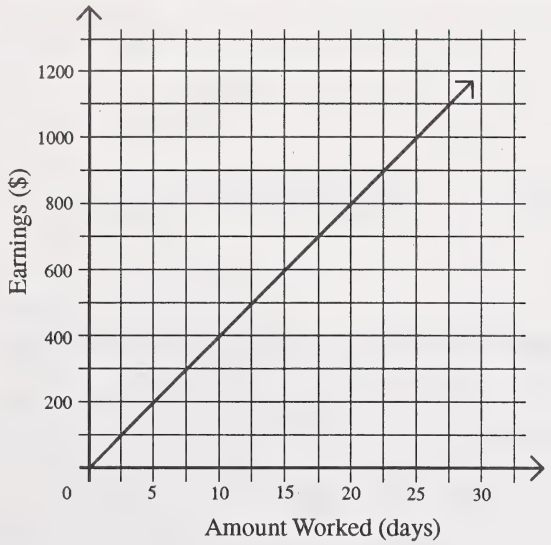
b. How many seconds will it take Donna to walk 48 m?

c. Make a table of values and draw the graph to show the relation.





The following graph applies to questions 3, 4, and 5.



3. Complete the following ordered pairs.

   a. (, 200) b. (12,)

   c. (21,) d. (4,)

   e. (, 440) f. (, 360)

4. Find the amount earned for the following days worked.

   a. 20 days b. 17 days

   c. $24\frac{1}{2}$ days d. 19 days

   e. 27 days

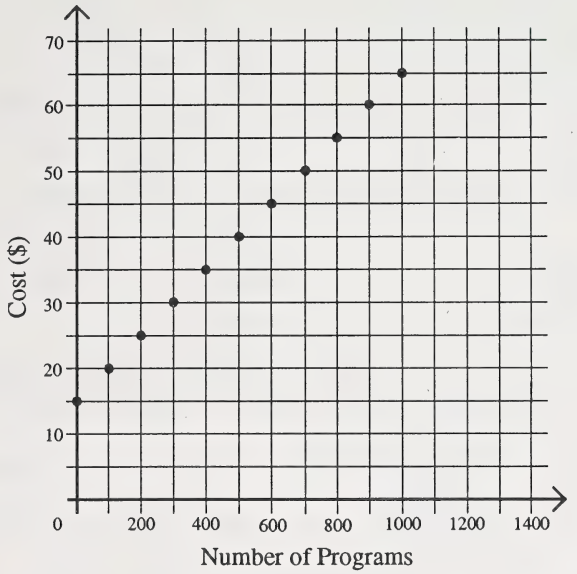
5. Determine how many days it would take to earn the following amounts.

   a. $600 b. $540

   c. $100 d. $1100

   e. $250

6. The following graph represents the relation between the number of programs printed for a school play and the cost of printing.



a. Find the cost of printing the following numbers of programs.

   i. 800 programs

   ii. 600 programs

   iii. 1300 programs

b. Determine how many programs can be printed for the following amounts.

   i. $25

   ii. $50

   iii. $95

c. Why is it necessary to have separate points on the graph?

d. Give a possible reason for the point (0, 15) on the graph.

For solutions to **Activity 2**, turn to the **Appendix**, **Topic 1**.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

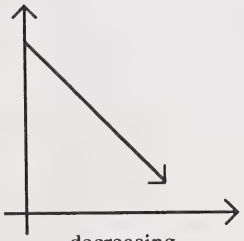
} You may decide to do both.

Extra Help

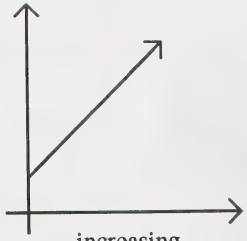
Graphing Relations

The following are important points to remember:

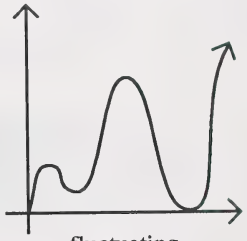
- A relation shows an association between ordered pairs.
- Graphing is an effective way of showing relations.
- Graphs show whether relations are decreasing, increasing, fluctuating, or remaining constant.



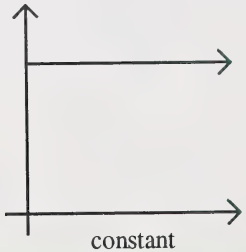
decreasing



increasing



fluctuating



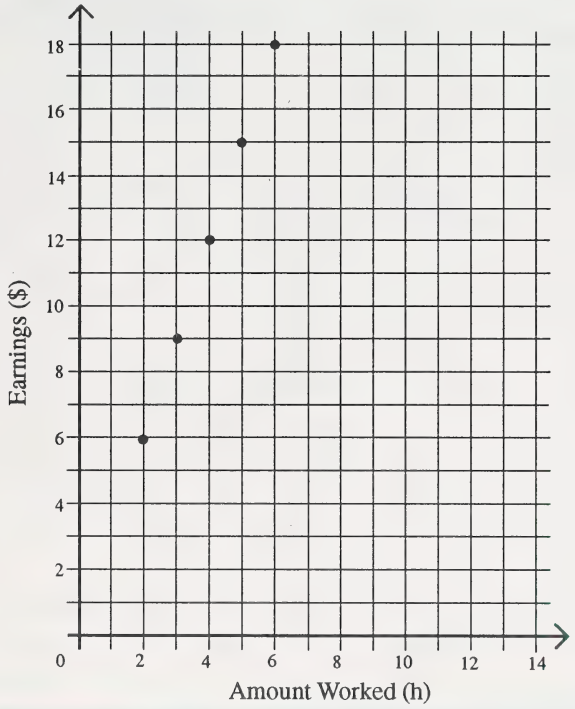
constant

- A relation can be shown by using a table of values. A table of values may look like the following:

Amount Worked (h)	2	3	4	5	6	7	8	9	10
Earnings ($)	6	9	12	15	18	21	24	27	30

- A table of values can be used to get ordered pairs. The ordered pairs from the previous table of values would be (2, 6), (3, 9), (4, 12), (5, 15), (6, 18), (7, 21), (8, 24), (9, 27), and (10, 30).

- The ordered pairs (2, 6), (3, 9), (4, 12), (5, 15), and (6, 18) can be plotted on a coordinate plane.

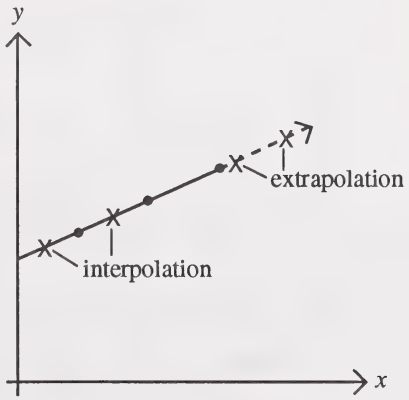


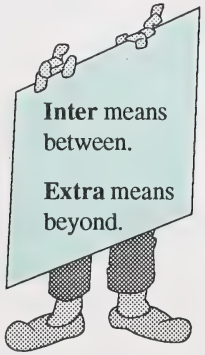
The horizontal axis represents the independent variable. The vertical axis represents the dependent variable. This means that in any ordered pair, the y-value depends on the x-value.

- Values which are not in the group of ordered pairs can be determined using interpolation or extrapolation.

  Interpolation is information taken from between existing points.

  Extrapolation is information taken from beyond existing points.



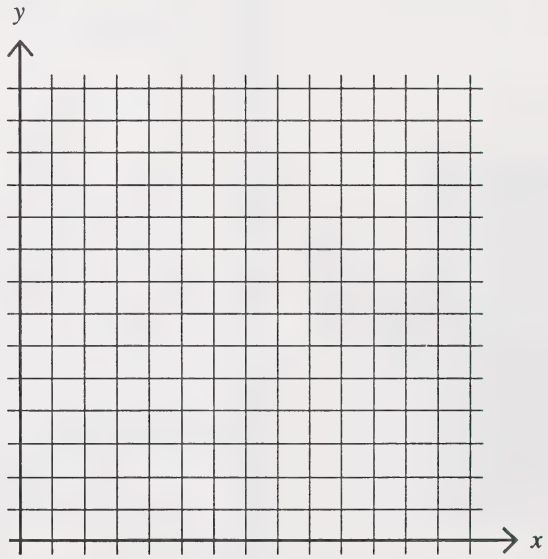


For more practice, do the following questions.

Determine the pattern of the relation in each of the following. Develop appropriate ordered pairs and plot the points on a coordinate plane. From the graph, interpolate and extrapolate other related values. Find two points for each.

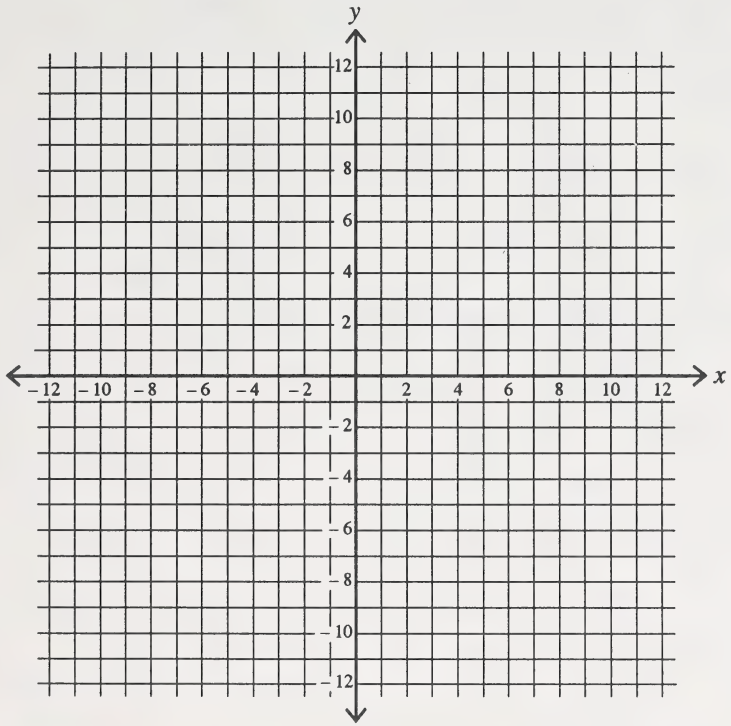
1.

x	0	1	2	3		8	10	
y	3	4		6	9			14



2.

x	0	2	3		7	8		12
y	−2	0	1	4			8	



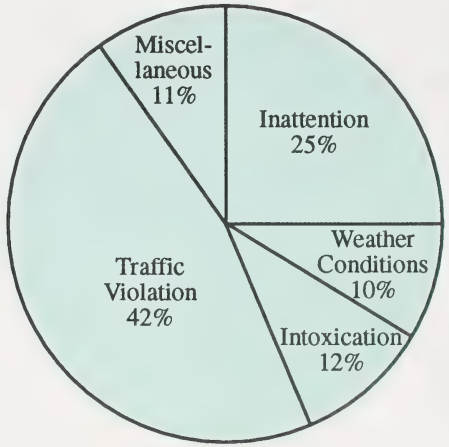
For solutions to **Extra Help**, turn to the **Appendix**, **Topic 1**.

Extensions

Relations among numbers can be illustrated using other kinds of graphs in addition to line graphs. One such graph is a circle graph. It is circular and looks like a pie that has been divided. Circle graphs are easy to read, and they show a great deal of information at a glance. Consider this example.

Example 12

Suppose this circle graph shows the causes of fatal automobile accidents. Calculate the number of fatalities by each cause if you were considering 900 fatal automobile accidents.



Solution:

Inattention $25\% \text{ of } 900 = 0.25 \times 900$
$= 225$

Traffic Violation $42\% \text{ of } 900 = 0.42 \times 900$
$= 378$

Intoxication $12\% \text{ of } 900 = 0.12 \times 900$
$= 108$

Weather Conditions $10\% \text{ of } 900 = 0.1 \times 900$
$= 90$

Miscellaneous $11\% \text{ of } 900 = 0.11 \times 900$
$= 99$

There are even more ways of showing relations. Study the following example which illustrates a fluctuating line graph.

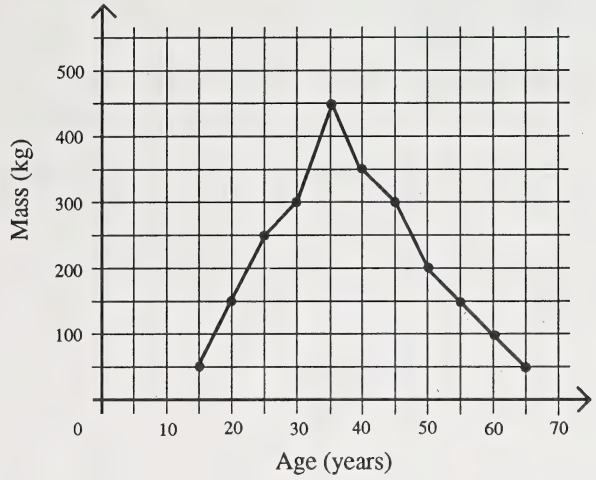
Example 13

The following table shows the mass in kilograms that a person could lift at different ages. Study the table; then answer the questions.

Age	15	20	25	30	35	40	45	50	55	60	65
Mass	50	150	250	300	450	350	300	200	150	100	50

- Make a line graph to represent the data in the table.

Solution:



- At what age was the person able to lift the heaviest weight?

Solution:

The person was able to lift the heaviest weight when about 35 years old.

Now try the following questions.

1. A grocery store made a total profit of $3000 in the month of July.

   a. Use this table to prepare a circle graph.

Departments	Percent of Total Profit
Grocery	41%
Produce	25%
Meat	20%
Bakery	8%
Sundries	6%

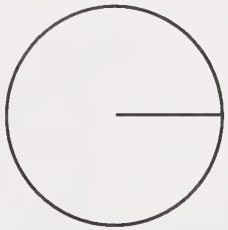
Hint: An entire circle is made up of 360°. To find the number of degrees for each section, proceed as shown for the grocery department. Round each calculation to the nearest degree.

Grocery: 41% of $360° = 0.41 \times 360°$

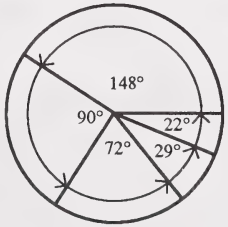
$= 147.6°$

$= 148°$

Begin the circle graph by drawing a radius as in the following diagram.



From this radius, draw the first section of 148°. Continue with each section until you get something like this.



b. Calculate the profit made by each department using the table in part a. Follow the example given for the grocery department.

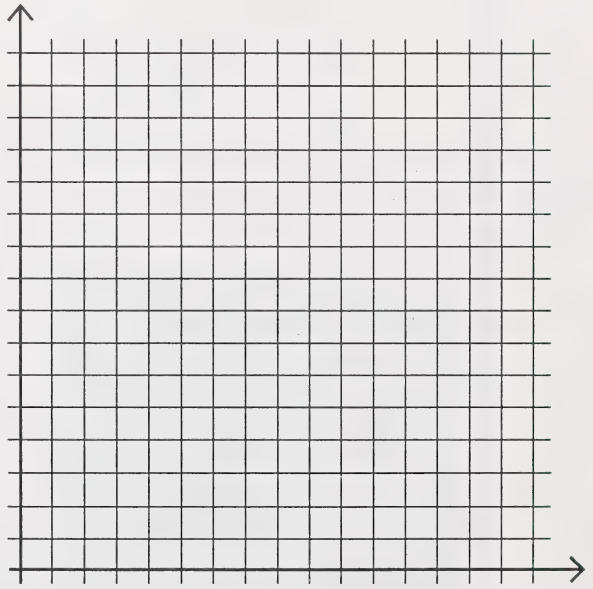
Grocery department: 41% of $\$3000 = 0.41 \times \$3000$

$= \$1230$

2. Speed is how fast a vehicle is travelling, and distance is how far it takes a vehicle to stop after applying the brakes.

a. Use the following table to make a graph.

Speed (km/h)	10	20	30	40	50	60	70	80
Distance (m)	4	8	12	16	20	24	28	32

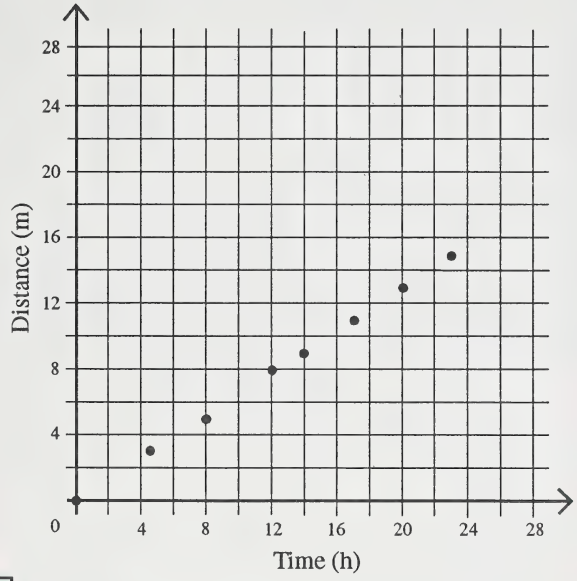


b. According to a graph, if a vehicle were going 45 km/h, what distance would it travel before coming to a stop after the brakes are applied?

c. A vehicle is travelling 75 km/h. The brakes are applied. What distance will the vehicle travel before it comes to a stop? Use interpolation.

d. If it took 40 m for a vehicle to stop after the brakes were applied, how fast was it travelling before the brakes were applied? Use extrapolation.

3. Study the following coordinate plane, and make a table of values for the points shown.

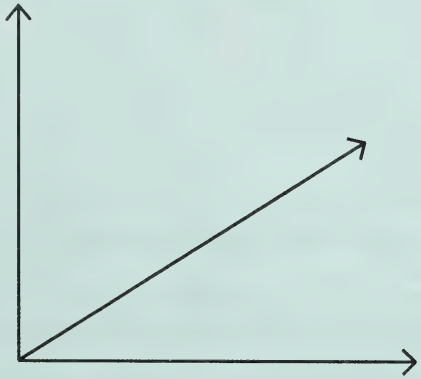


For solutions to **Extensions**, turn to the **Appendix, Topic 1**.

Topic 2 Direct Variations

Introduction

Some relationships between two variables are straight forward. When one quantity increases, the other increases. When one quantity decreases, the other decreases. These relationships are known as direct variation. It can be said that the two quantities vary directly. A direct variation is always a straight line graph as shown. One value is directly proportional to the other value.



What Lies Ahead

Throughout the topic you will learn to

1. calculate the constant, graph equations, and solve problems related to direct variation

Now that you know what to expect, turn the page to begin your study of direct variations.

Exploring Topic 2

Activity 1

Calculate the constant, graph equations, and solve problems related to direct variation.

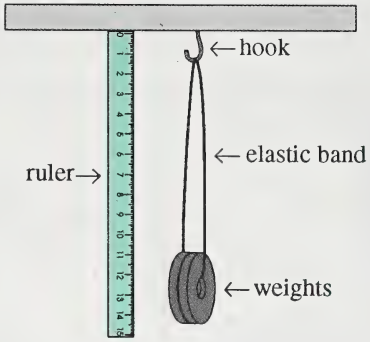
In the previous topic you were taught the skills needed to graph relations and interpret data. You will now examine a special kind of relation called **direct variation**.

To get a feel for direct variation, consider the following.

- The more you work, the more you earn.
- The more candies you eat, the greater chance you have of getting cavities.
- The more you exercise, the more you sweat.
- The more cigarettes you smoke, the greater chance you have of developing lung cancer.

Two variables do not always vary directly. You will deal with other types of variations in **Topic 3**.

Some students performed an experiment in which they placed various weights, one at a time, on an elastic band. The elastic band supported the weights. After each weight was placed on the band, the students measured the length of the stretched band. With heavier weights, the elastic band stretched more.



Only the amount of **stretch** in the band was measured.

The following table shows their findings.

Stretch Length (cm)	0	2.2	3	4.3	5	6.5	7
Weight (kg)	0	6.6	9	12.9	15	19.5	21

This experiment is an example of direct variation.

For the duration of the experiment, you could say the following:

- The length of the stretched elastic band varies directly with the amount of weight.
- The length of the stretched elastic band is directly proportional to the weight.

In most relations, a change in one variable affects the other variable in a predictable way. You can use an equation to describe this relation.

Consider the experiment with the elastic band.

Notice that the length of the elastic band is equal to a given number times the weight.

This can be written mathematically as $l = k \times w$, where k is a number. Since k always remains the same, it is called a **constant**.

From time to time, you will be asked to find the value of the constant k.

You can see from the table of values that if you applied a weight of 9 kg, the band would stretch 3 cm.

Use the formula $l = k \times w$.

Substitute the values you know in $l = k \times w$, and calculate k.

$$l = k \times w$$
$$3 = 9 \times k$$
$$3 = 9k$$
$$\frac{3}{9} = \frac{9k}{9}$$
$$k = \frac{1}{3}$$

The constant k equals $\frac{1}{3}$.

You can use the value of k to find the relationship between l and w.

Substitute the value of k in the formula $l = k \times w$.

You get $l = \frac{1}{3}w$. This can be changed to $w = 3l$ by multiplying both sides of the equation by 3.

$$l = \frac{1}{3}w$$
$$3 \times l = 3 \times \frac{1}{3} \times w$$
$$3l = w$$
$$w = 3l$$

The weight is always three times the stretched length of the elastic band.

If the value of l increases, then the value of w must also increase. If the value of l decreases, then the value of w must also decrease.

For example, look at the following:

Let $l = 100$.
$w = 3 \times 100$
$= 300$

Let $l = 2$.
$w = 3 \times 2$
$= 6$

When l decreased from 100 to 2, then w decreased from 300 to 6.

This shows the relationship between the two variables in a direct variation.

Look at other examples of direct variation.

- $y = 3x$
  The y-value is always 3 times as great as the x-value. The value of k is 3.

- $C = \frac{1}{2}h$
  The C-value is always $\frac{1}{2}$ the h-value. The value of k is $\frac{1}{2}$.

- $M = -2n$
  The M-value is always -2 times the n-value. The value of k is -2.

You should now be able to identify direct-variation equations and use graphing techniques to graph direct variation. As you go along, you will discover an interesting pattern for graphs which show direct variation.

In the next example you will see how to draw a graph of an equation which represents direct variation.

Example 1

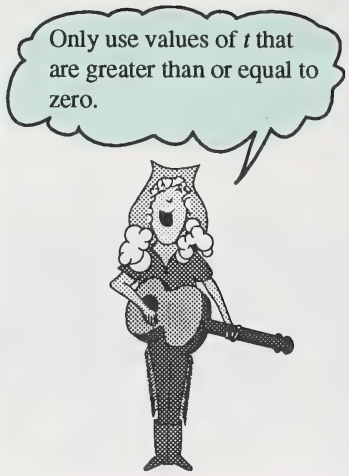
Given the equation $h = 3t$, draw the graph. In this equation, h is the dependent variable since it has to be calculated from a known value of t, which is the independent variable.

Solution:

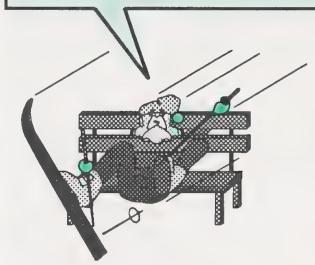
First you need a table of values. Since you do not have any actual values for t, you can choose any values. In this example, values of 0, 1, 2, 3, 4, and 5 were chosen for t. The values for h are calculated from the relation $h = 3t$.

When $t = 0$, $h = 3t$
$= 3 \times 0$
$= 0$

When $t = 1$, $h = 3t$
$= 3 \times 1$
$= 3$



You must remember that the dependent variable is always on the vertical axis and the independent variable is always on the horizontal axis. Mixing these up would make a graph go astray.



When $t = 2$, $h = 3t$

$= 3 \times 2$

$= 6$

When $t = 3$, $h = 3t$

$= 3 \times 3$

$= 9$

When $t = 4$, $h = 3t$

$= 3 \times 4$

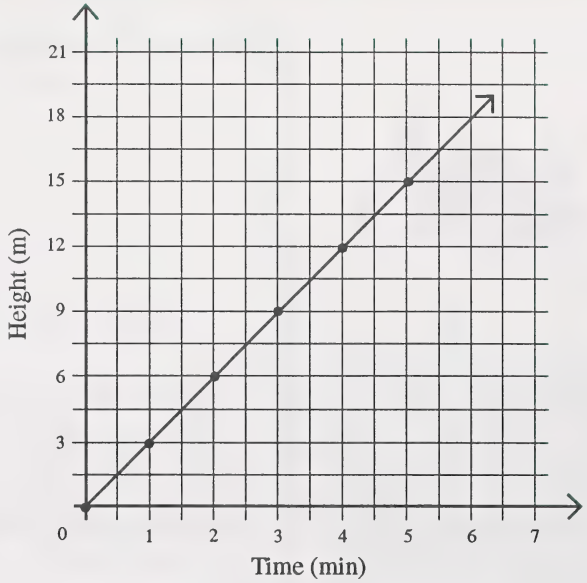
$= 12$

When $t = 5$, $h = 3t$

$= 3 \times 5$

$= 15$

t	0	1	2	3	4	5
h	0	3	6	9	12	15



As stated previously, h is the dependent variable. It could represent height above sea level as a person walks up a slope. The variable t is the independent variable, and it could represent the time that goes by as the person ascends the slope. Once a pattern is established, additional points can be plotted because of the direct-variation relationship, whether or not the ordered pairs are known. Direct variation, when graphed, is always a straight line.

You know how to graph from a table of values or from an equation. You will now use a table of values or a graph to determine an equation that describes a direct-variation relation.

Example 2

The following table of values shows the distance travelled by a car which is moving at a constant speed.

Time (t) (h)	0.5	1	1.5	2	2.5	3
Distance (d) (km)	45	90	135	180	225	270

- Use the previous table to construct a graph showing the relation between distance travelled and time.

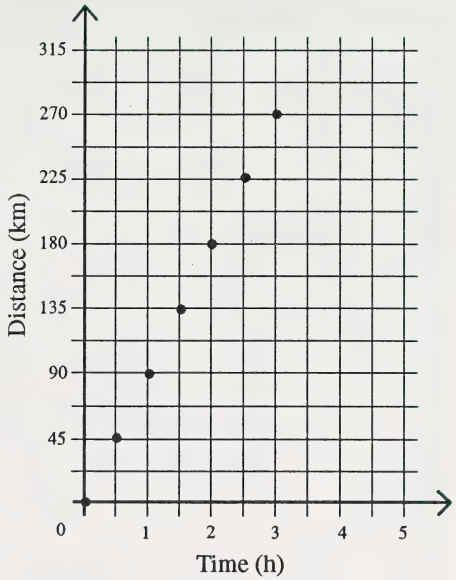
To find k, use $k = \frac{\text{dependent variable}}{\text{independent variable}}$.

If $\frac{d}{t} = k$, then $t \times \frac{d}{t} = t \times k$

$d = tk$

$d = kt$

Solution:



- Develop the equation from the relation.

Solution:

If you calculate the ratio $\frac{\text{dependent variable}}{\text{independent variable}}$ or $\frac{d}{t}$ for several ordered pairs, you will get an interesting result.

In a relation such as $d = kt$, k is the constant of variation. Once the k-value is known for any particular relation, this value does not change.

Study the following ratios of ordered pairs.

$$\frac{45}{0.5} = 90 \qquad \frac{90}{1} = 90$$

$$\frac{180}{2} = 90 \qquad \frac{225}{2.5} = 90$$

$$\frac{270}{3} = 90$$

Observe that you always get the same value of 90 when each ratio is simplified.

This value is called the **constant of variation**.

If you call distance d and time t, then the previous relationship can be expressed as $\frac{d}{t} = k$. Thus, k is the constant of variation.

The equation $\frac{d}{t} = k$ can be written as $d = kt$ and since $k = 90$, the equation becomes $d = 90t$. The equation $d = 90t$ describes the previous table of values.

Notice that the graphs for direct variation always begin at the origin (0, 0).

Now study this example.

Example 3

Wendy works at the Parkside Grocery Store and is paid by the hour. Her earnings over five weeks are shown in the following table.

Week	1	2	3	4	5
Hours worked (h) (h)	4	8	12	16	20
Earnings (e) ($)	32	64	96	128	160

- Determine the constant of variation and the equation that defines this relation.

Solution:

$$k = \frac{\text{dependent variable}}{\text{independent variable}}$$

$$\therefore k = \frac{\text{earnings}}{\text{hours}}$$

$$= \frac{e}{h}$$

Substitute ordered pairs from the table.

$$k = \frac{32}{4} \text{ or } k = \frac{64}{8}$$

$$= 8 \qquad = 8$$

$a = kb \leftarrow$ independent variable
$\uparrow$
dependent variable

The dependent variable equals the constant times the independent variable. The letter k is usually used to stand for the constant of variation.

Note that k was calculated twice to see if it was constant. The answers were equal; therefore, $k = 8$. Since $k = \frac{e}{h}$ and $k = 8$, then $\frac{e}{h} = 8$ or $e = 8h$.

- If Wendy worked 29 h, what would her earnings be?

Solution:

If you know how many hours Wendy worked, the equation $e = 8h$ can be used to calculate Wendy's earnings.

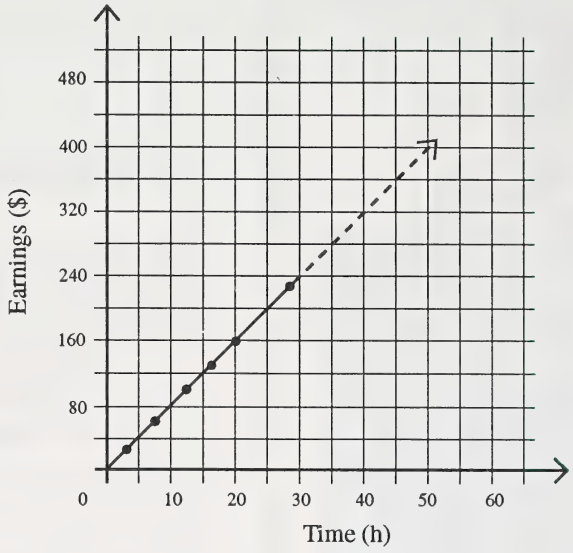
To calculate Wendy's earnings for 29 h of work, you need to extrapolate beyond the readings in the table of values.

$$\begin{aligned} e &= 8h \\ &= 8 \times 29 \\ &= 232 \end{aligned}$$

Wendy would earn \$232 in 29 h.

- Draw a graph showing Wendy's earnings.

Solution:



- If Wendy earns \$392, calculate how many hours she worked to earn the money.

  Solution:

  If you know Wendy's earnings, you can calculate how many hours she actually worked by using the same equation $e = 8h$.

$$e = 8h$$
$$392 = 8 \times h$$
$$8h = 392$$
$$h = \frac{392}{8}$$
$$= 49$$

  Wendy worked 49 h to earn \$392.

- According to the graph, how many hours did Wendy work to earn \$392?

  Solution:

  According to the graph, Wendy earned \$392 in about 49 h. Estimation is needed to find 49 h.

Now that you are able to identify the equation for a direct variation, you can apply these skills in solving problems of direct variation.

You will be shown a different way to calculate the constant of variation and find the equation for that relation.

The statement ***a* varies directly as *b* varies** can be written as $a \propto b$. If $a \propto b$, then $a = kb$.

Look at the following examples to see how you would use these two ideas in solving problems.

The expression $a \propto b$ can be read as

- a varies directly as b varies
- a varies as b varies
- a is proportional to b
- a is directly proportional to b

The statement $A \propto l$ means that the area (A) varies directly as the length (l) varies.

Example 4

The area of a carpet varies directly as the length varies. When the area is 15 m^2, the length is 7.5 m. Find the length when the area is 20 m^2.

Solution:

Use $A \propto l$, where A represents the area and l represents the length.

$A = 15$ and $l = 7.5$
(This information is given in the question.)

$$A = kl$$
$$15 = k \times 7.5$$
$$k \times 7.5 = 15$$
$$\frac{k \times 7.5}{7.5} = \frac{15}{7.5}$$
$$k = \frac{15}{7.5}$$
$$= 2$$

Substitute $k = 2$ in $A = kl$, which is the equation for this relation.

Therefore, $A = 2l$.

Use $A = 2l$ to solve for l when $A = 20$.

$$A = 2l$$
$$20 = 2l$$
$$2l = 20$$
$$\frac{2l}{2} = \frac{20}{2}$$
$$l = 10$$

The length is 10 m when the area is 20 m^2.

Example 5

If it costs \$5 000 000 to construct 8 km of railroad, how much will it cost to construct 60 km of railroad?

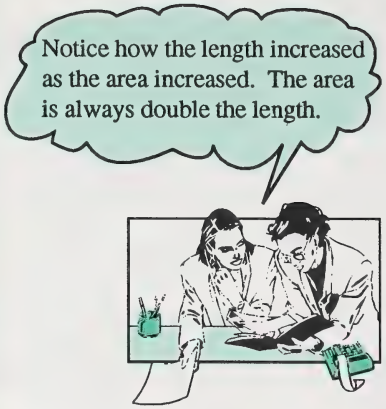
Solution:

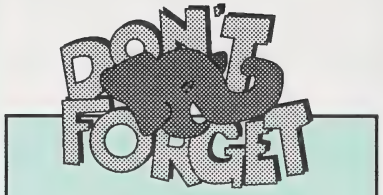
Let E be the expense and l be the length.

$$E \propto l$$
$$E = kl$$
$$5\,000\,000 = k(8)$$
$$k = 625\,000$$

Substitute $k = 625\,000$ in $E = kl$ to find the defining equation.

The defining equation is $E = 625\,000l$.





- The area of a rectangle is given by $A = l \times w$ or $A = lw$.
- When a varies directly as b varies, this relation is written as $a \propto b$ or $a = kb$.

For 60 km, the cost will be as follows:

$$E = 625\,000 \times 60$$
$$= 37\,500\,000$$

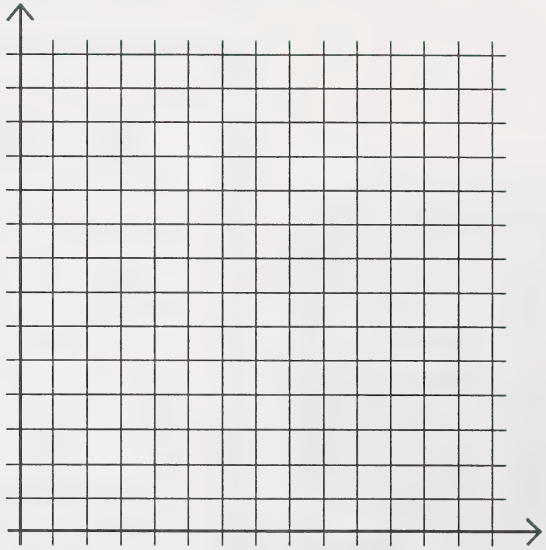
The cost to construct 60 km of railroad is $37 500 000.

Do at least four of the following questions.

1. During the summer, Rachel works as a lifeguard at Sirgany's Beach. She is paid by the hour. The following table shows her earnings for the number of hours worked over a period of time.

Amount Earned ($)	30	60	90	120	150	180	210
Time (h)	5	10	15	20	25	30	35

Use the table to make a graph showing the relationship between the hours worked by Rachel and the amount she earned.



2. Which of the following describe direct variation?

   a. The more you pump air into a tire tube, the weaker it gets.

   b. The shorter a pole is, the smaller its shadow will be.

   c. The more you turn on a tap, the more water will flow.

   d. The greater the flow of water, the less time it takes to fill a container.

   e. The faster you run, the sooner you get there.

f. The nearer a torch is to you, the brighter the light will be.

g. The harder you work, the more pay you will receive.

h. The higher the voltage is, the greater the resistance will be.

i. The larger the radius is, the bigger the circle will be.

3. The following tables show direct variations. Find the constant of variation. In each case, use k as the symbol for the constant of variation.

a.

Time (h)	3	4	6	10	13
Earnings ($)	15	20	30	50	65

b.

Weight (kg)	4.5	6	9	10.5	13.5	15
Length (cm)	3	4	6	7	9	10

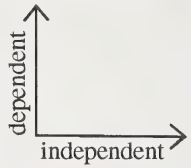
4. Graph each relation, and give the equation that defines the relation.

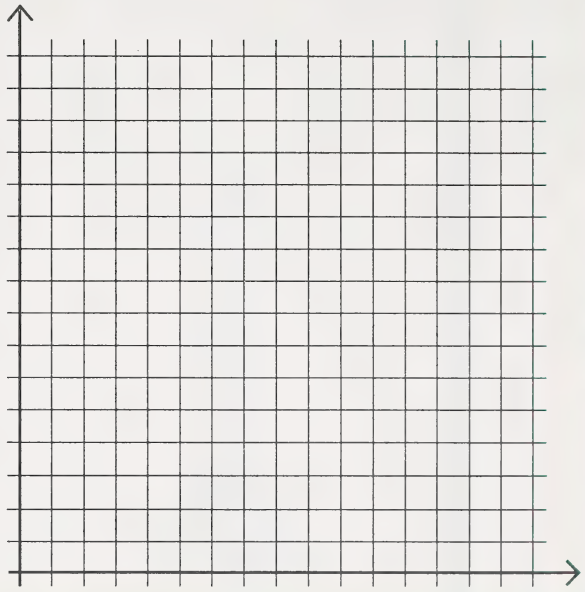
a. $V \propto I$

Current (I)	2	6	10	14	18	22	26
Voltage (V)	3	9	15	21	27	33	39

The dependent variable varies directly as the independent variable varies. The expression $V \propto I$ means that the voltage (V) will be on the vertical axis and the current (I) will be on the horizontal axis.

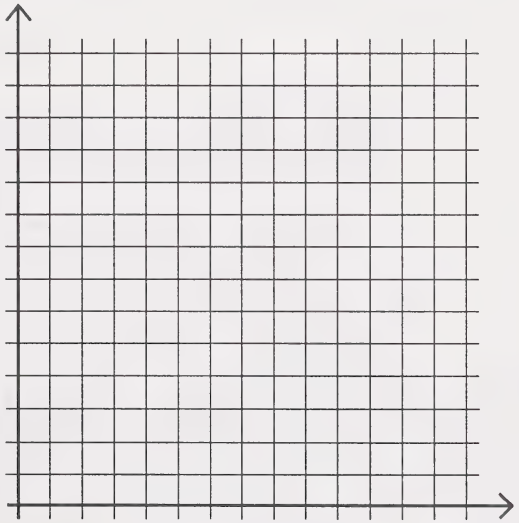
Recall which variable is placed on which axis:





b. $A \propto l$

Length (cm)	0	7	8	9	10	11	12	14	15	22	25
Area $\left(\text{cm}^2\right)$	0	49	56	63	70	77	84	98	105	154	175



5.

Current (I)	2	6	10	14	18	22	26
Voltage (V)	3	9	15	21	27	33	39

a. Find the voltage when the current is as follows.

i. 70 amperes ii. 10 amperes iii. 105 amperes

b. Find the current when the voltage is as follows.

i. 150 volts ii. 63 volts iii. 20 volts

6.

Length (cm)	0	7	8	9	10	11	12	14	15	22	25
Area (cm^2)	0	49	56	63	70	77	84	98	105	154	175

a. Find the area when the length is as follows.

i. 14 units ii. 16 units iii. 30 units

b. Find the length when the area is as follows.

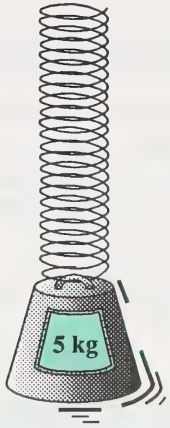
i. 175 square units ii. 217 square units iii. 42 square units

7. The length of the extension of a spring varies directly as the weight attached to it varies. If the spring is stretched 2.5 cm by a weight of 5 kg, answer the following.

a. Determine the equation of the variation.

b. Find the extension when a weight of 30 kg is applied.

c. Find the weight when the spring has an extension of 56 cm.



For solutions to **Activity 1**, turn to the **Appendix, Topic 2**.

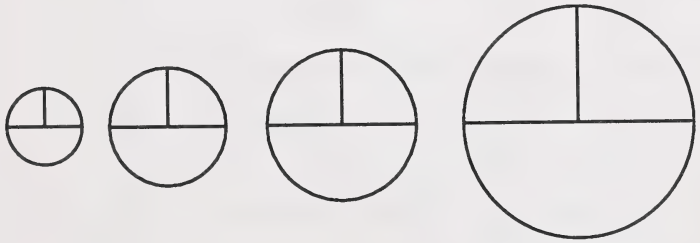
If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section. } You may decide to do both.

Extra Help

Direct variation shows how two variables increase or decrease. Two common variables are time and distance. If you are walking along a road, you should find that the longer you walk, the further you go. You might say everybody knows this, but you need to consider this if you want to make a graph or find the constant of variation. First, you need to know that when two quantities vary directly, there is a constant behaviour. Second, you should be able to put this down on paper and draw the graph.

Look at the following circles. Consider the radius of each, and then consider each diameter.



From the circles, you should see that the following relations are true.

- the bigger the circle, the bigger the radius
- the bigger the circle, the bigger the diameter
- the smaller the circle, the smaller the radius
- the smaller the circle, the smaller the diameter

In the following example, values are given to the variables.

Example 6

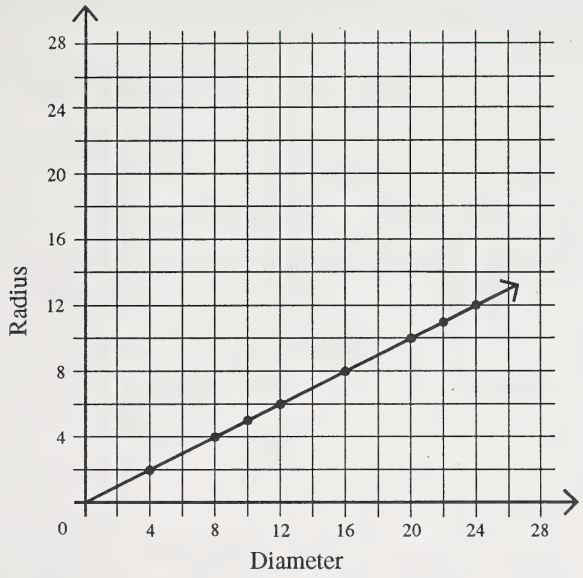
The relation between the radius of a circle and the diameter is given by $r = \frac{1}{2}d$. Construct a graph for this relation.

Solution:

First make a table of values.

Diameter (d)	4	8	10	12	16	20	22	24
Radius (r)	2	4	5	6	8	10	11	12

Then plot the points on a grid.



Look at this example.

Example 7

John has a box of pennies, but he does not have the time to count them one at a time. Instead, he makes piles of seven pennies. Each pile of pennies measures 1 cm.

- How much money would John have if the pile were 0.1 m high?

  Solution:

  For every 1 cm there are seven pennies.
  In 1 m there are 100 cm; thus, in 0.1 m there are 10 cm.

  Therefore, the number of pennies is $10 \times 7 = 70$.

  If the pile were 0.1 m high, John would have 70 pennies.

- How high would a pile worth \$0.21 be?

  Solution:

  Note that \$0.21 is equal to 21 pennies.

  Therefore, the number of centimetres is $21 \div 7 = 3$.

  A pile of \$0.21 would be 3 cm high.

- Make a table showing the direct variation.

  Solution:

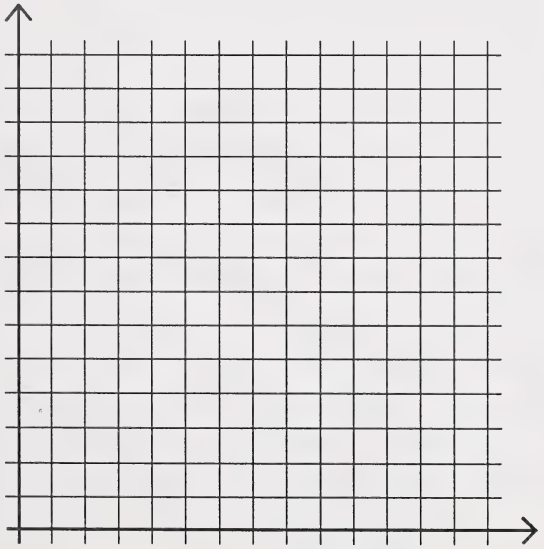
Centimetres	1	2	5	8	11	15	20	50
Cents	7	14	35	56	77	105	140	350

Try some of these questions.

1. The following table of values describes a case of direct variation.

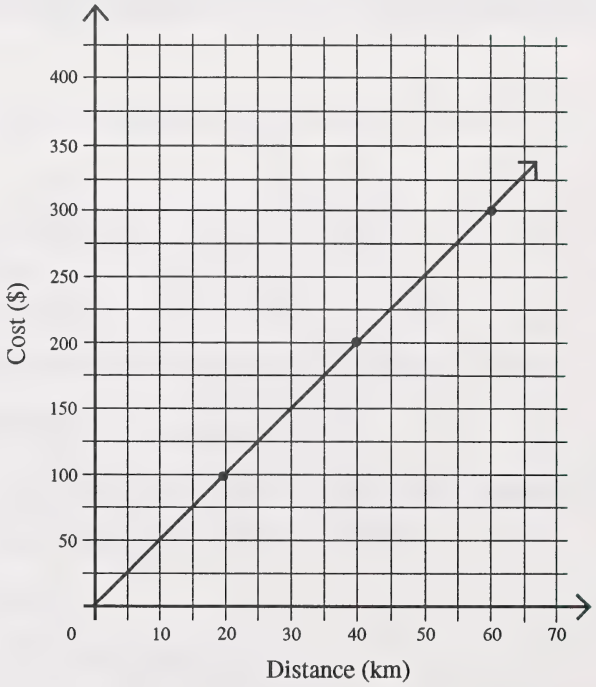
Length (l) (cm)	1	2	3	4	5
Weight (w) (kg)	4	8	12	16	20

a. Draw the graph for this table of values.



b. What is the constant of variation?

2. The following graph represents the relation between the distance travelled and the cost of a boat ride.



Use the graph to answer the following.

a. Find the cost of travelling the following distances.

   i. 20 km ii. 40 km

   iii. 44 km iv. 60 km

   v. 70 km

b. Find the distance travelled for the following amounts of money.

   i. $160 ii. $180

   iii. $240 iv. $280

   v. $350

For solutions to **Extra Help**, turn to the **Appendix**, **Topic 2**.

Extensions

Direct variation can always be represented graphically by a straight line or algebraically as a linear relation.

If two variables are related in a direct way, the expression linking them is an equation.

This equation can be plotted in graph form as a straight line. For direct variation, the graph always passes through the origin. It takes the form $y = ax$, but it never takes the form $y = ax + b$, which other linear relations may take.

When the variation is established as being direct, the value of the constant of variation does not change.

When the variables are given as x and y, this generally implies that the units are not of particular importance. The variables used for direct variations are more often than not applicable to practical situations.

For the general equation $y = ax$, x and y are the variables and a is the constant of variation.

For practical purposes, you could consider the case of $w = kl$, where w is weight in kilograms, l is length in centimetres, and k is the constant of direct variation.

The spring balance is based on this principle. You can determine the value of k through experimentation by placing known weights on the balance and then measuring l. When k is known, you can develop a table of values as shown.

l	1.5	2	2.5	3.5	4	5	6.5	7
w	4.5	6	7.5	10.5	12	15	19.5	21

What is k in this table? The value of k is found by using the following formula.

$$w = kl$$

$$k = \frac{w}{l}$$

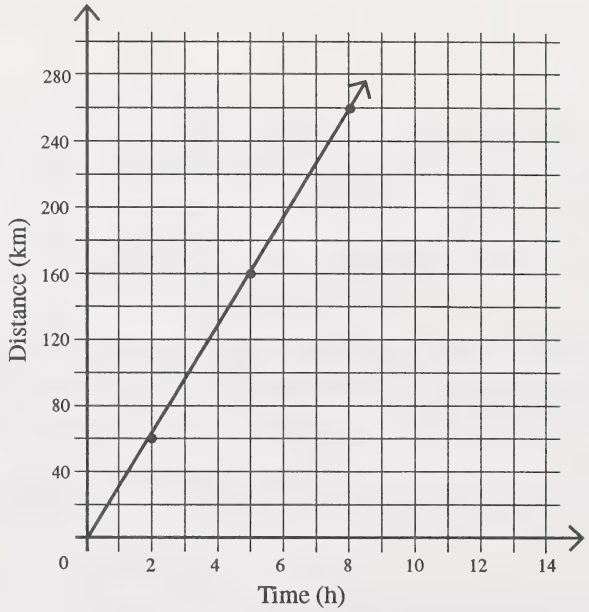
$$= \frac{4.5}{1.5}$$

$$= 3$$

When the table of values is known, you can complete a graph using those values. Then you can extrapolate and interpolate to answer a whole variety of questions.

Now try a few more questions relating to direct variation.

The following graph represents a relation between the distance travelled by bus and the amount of time spent travelling.



1. Determine the distance the bus travelled in the following lengths of time.

   a. 2 h

   b. 3.2 h

   c. 6.2 h

   d. 8 h

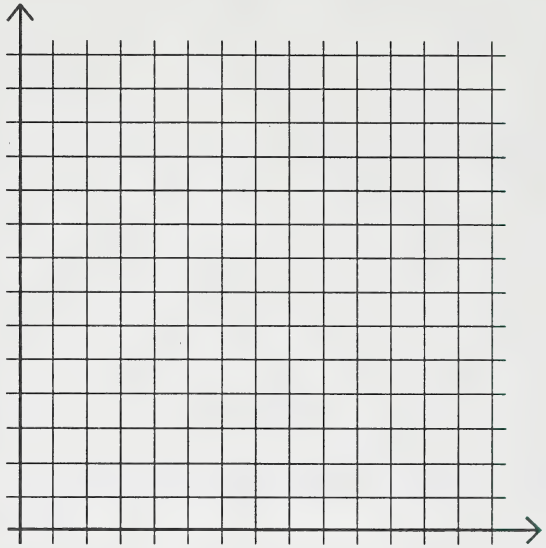
   e. 8.8 h

2. Refer to the previous graph. Find the time it would take to travel the following distances.

   a. 80 km
   b. 120 km
   c. 180 km
   d. 240 km
   e. 300 km

3. If the bus travels 300 km, calculate its average speed. Use the time you obtained for question 2e.

4. The equation $d = 8L$ relates the number of kilometres travelled to the number of litres of gasoline used. Make a table and draw a graph for this relation.



For solutions to **Extensions**, turn to the **Appendix**, **Topic 2**.

Topic 3 Inverse and Partial Variations

Introduction

Not all relations are direct. In this topic you will be introduced to inverse variations and partial variations.

Historically, extensive work was done on variation by people such as Fourier, La Place, Mellin, and La Grange. An example of inverse variation is that the longer you drive your car, the less gasoline you will have in your gas tank.

When preparing an estimate for a project, contractors sometimes use a fixed charge plus a variable charge. Partial variation is used to calculate the total charge. You will learn how this is done in this topic.

What Lies Ahead

Throughout the topic you will learn to

1. graph and solve problems leading to inverse variation

2. graph and solve problems leading to partial variation

Now that you know what to expect, turn the page to begin your study of inverse and partial variations.

Exploring Topic 3

Activity 1

Graph and solve problems leading to inverse variation.

Not all situations fit the requirements for direct variation. There are other forms that variation can take. You will now look at another form of variation called inverse variation.

In the last topic there were always two variables in the direct-variation equation. When one variable increased, so did the other variable. When one variable decreased, the other variable decreased.

This is not the case for inverse variation. Instead, the following applies. When one variable increases, the other variable decreases; or when one variable decreases, the other variable increases.

Here is an example of inverse variation.

Three men take ten days to paint Northlands Coliseum. Six men can do the same job in five days.

The variables are the number of men and the number of days. Take note of how these variables are related. Notice that as the number of men increases, the number of days decreases.

Here is another case of inverse variation.

When you jog at a rate of 3 km/h, you can jog 12 km in 4 h. If you decide to jog at a rate of 6 km/h, the 12 km can be covered in 2 h.

The variables are speed and time. As you can see, if you increase your speed, then your time automatically decreases.

Study the following table to see that as the first value is decreasing at a steady rate, the second value is increasing at the same steady rate.

80	40	20	16	10	8	5	4	2
1	2	4	5	8	10	16	20	40

Using the table, you can say that as the top value decreases, the bottom value increases; or as the bottom value increases, the top value decreases.

You may summarize these two statements by saying that the top value **varies inversely** as the bottom value varies, or the bottom value varies inversely as the top value varies.

By examining the table even closer, you may notice that when the two values are multiplied, the result is the same product.

$80 \times 1 = 80$

$40 \times 2 = 80$

$20 \times 4 = 80$

$16 \times 5 = 80$

$10 \times 8 = 80$

$8 \times 10 = 80$

$5 \times 16 = 80$

$4 \times 20 = 80$

$2 \times 40 = 80$

You will see later that this product is the constant of variation. The constant of variation in this case is 80.

Since the product is the same for each pair of values, the rate of change is the same. This is what happens consistently when there is inverse variation. The constant of variation is the same in each case.

Consider another example of inverse variation. Notice the shape of the graph for this type of variation.

If the area of a rectangle is 80 cm^2, the dimensions could be as follows.

- It could be that the length is 80 cm and the width is 1 cm.
- It could be that the length is 40 cm and the width is 2 cm.
- It could be that the length is 20 cm and the width is 4 cm.
- It could be that the length is 16 cm and the width is 5 cm, and so on.

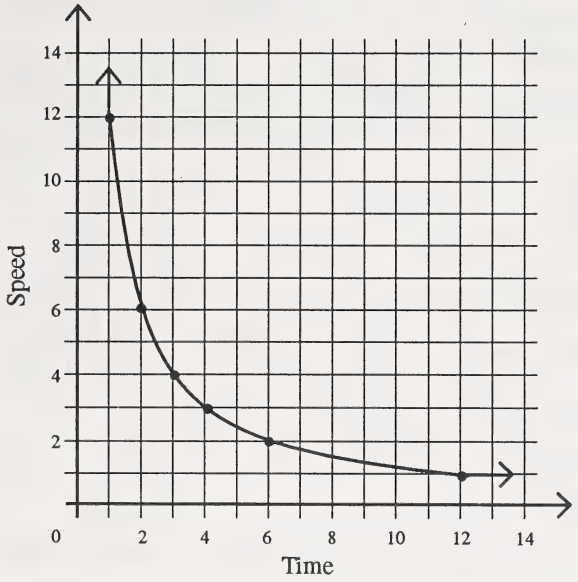
Example 1

- Use the following table to construct a graph.

Time (t)	1	2	3	4	6	12
Speed (s)	12	6	4	3	2	1

The dependent variable is always located on the vertical axis. In this case speed is the dependent variable and time is the independent variable.

Solution:



speed
↓
$d = st$ ← time
↑
distance

If the distance travelled is 12 km, the speed and the time could be as follows:

- It could be that the speed is 12 km/h and the time is 1 h.
- It could be that the speed is 6 km/h and the time is 2 h.
- It could be that the speed is 4 km/h and the time is 3 h.
- It could be that the speed is 3 km/h and the time is 4 h.

- What is the constant of variation?

Solution:

$\text{speed} \times \text{time} = k$ (k is the constant of variation.)

$s \times t = k$

$st = k$

$k = st$

From the table, $s = 6$ when $t = 2$.

$st = k$

$6(2) = k$

$12 = k$

$k = 12$

It is advisable to take more than one pair of values from the table since they all should have the same constant.

From the table, $s = 4$ when $t = 3$.

$st = k$

$4(3) = k$

$12 = k$

$k = 12$

The constant of variation is 12.

- Is the graph a straight line?

Solution:

No, it is a curved line called a hyperbola.

You have been shown the relationship between the two variables in an inverse relation.

Now you will be shown how to derive equations that describe inverse variation.

Example 2

The statement ***a* varies inversely with *b*** can be written as $a \propto \frac{1}{b}$.

If $a \propto \frac{1}{b}$, then $ab = k$.

- Find the equation that describes the inverse relation that was discussed in Example 1.

Solution:

Since speed varies inversely as time varies, you can write $s \propto \frac{1}{t}$.

$\therefore st = k$

To find the constant of variation for an inverse variation, multiply the dependent variable by the independent variable. For example, if $s = \frac{k}{t}$, then $k = st$.

Do you remember the following?

- The symbol $\propto$ means proportional.
- The symbol k is the constant of variation.

You know that when $s = 4,\ t = 3$.
Solve for k.

$$st = k$$
$$4(3) = k$$
$$12 = k$$
$$k = 12$$

Therefore, $st = 12$ is the equation that describes this relation.

- Use this equation to find the speed when $t = 2$.

Solution:

Substitute $t = 2$ in $st = 12$.

$$st = 12$$
$$s(2) = 12$$
$$2s = 12$$
$$\frac{2s}{2} = \frac{12}{2}$$
$$s = 6$$

The speed is 6 km/h. This answer agrees with the information in the table and on the graph.

At this stage you can safely make a statement such as the following:

Any equation that can be written in the form $k = st$ is an inverse variation.

Other examples of inverse-variation equations are $xy = 6,\ st = 50,$ and $mn = 1$.

Compare direct and inverse variation using a and b.

Direct Variation	Inverse Variation
$a \propto b$	$a \propto \frac{1}{b}$
$\therefore a = kb$	$\therefore ab = k$
$k = \frac{\text{dependent variable}}{\text{independent variable}}$	$k =$ dependent variable $\times$ independent variable

You have learned to identify the equation for an inverse variation. Now you will examine another graph of inverse variation.

Example 3

Graph the inverse variation $y = \frac{6}{x}$. You may want to rewrite $y = \frac{6}{x}$ as $xy = 6$.

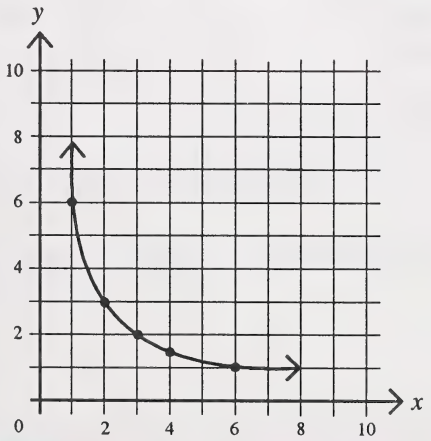
Solution:

First set up a table of values. Assign values to x and find the corresponding y-values.

x	1	2	3	4	6
y	6	3	2	1.5	1

For $y = \frac{6}{x}$ to be true, $x \neq 0$ since division by zero is undefined.

The ordered pairs are (1, 6), (2, 3), (3, 2), (4, 1.5), and (6, 1).



The graph for an equation of the form $y = \frac{6}{x}$ or $xy = 6$ is not a straight line. The general form for this type of equation is $a = \frac{1}{b}$ or $ab = k$.

Now that you can identify and graph inverse variations, study how to derive the equation for an inverse variation.

Example 4

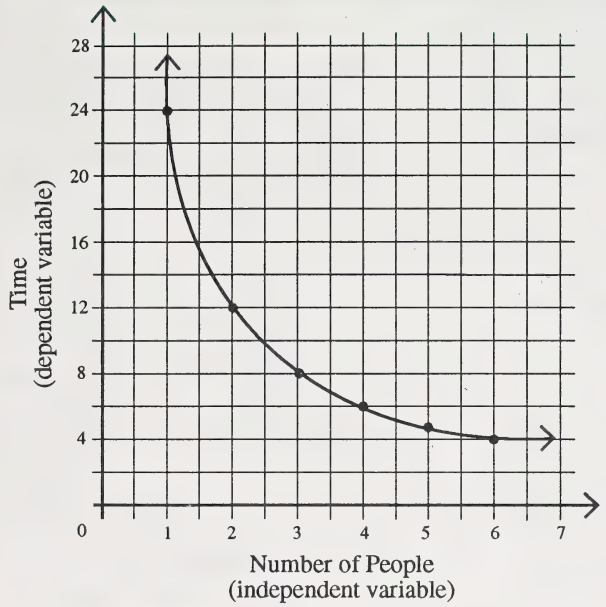
The number of hours needed to paint a house varies inversely as the number of people painting varies. This relation is shown by the following table.

Number of People (n)	1	2	3	4	5	6
Time (t)	24	12	8	6	4.8	4

The ordered pairs are (1, 24), (2, 12), (3, 8), (4, 6), (5, 4.8), and (6, 4).

Graph the relation defined by the table of values, and determine the equation of the variation.

Solution:



Since the time (t) varies inversely as the number of people (n) varies, you can write $t \propto \frac{1}{n}$ which implies that $nt = k$.

Now take one ordered pair from the graph or the table of values and substitute those values for t and n. This allows you to solve for k.

For example, choose $n = 4$, and $t = 6$.

$$nt = k$$
$$4(6) = k$$
$$24 = k$$
$$k = 24$$

$$\therefore nt = 24$$

The equation is $t = \frac{24}{n}$.

In the following example you will see how to find the equation of an inverse variation.

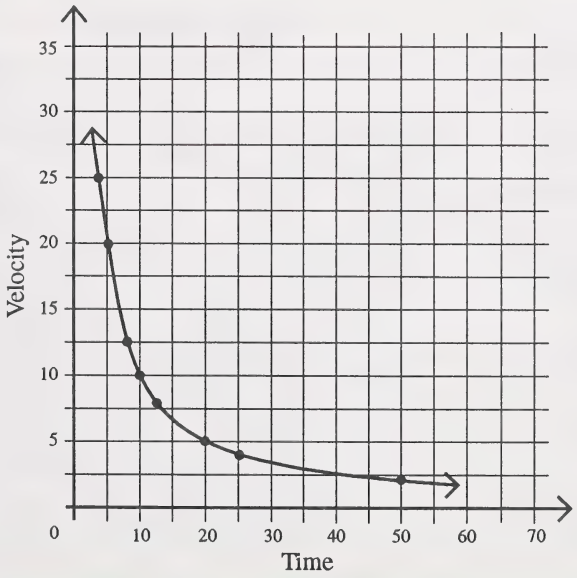
Example 5

The velocity of a fast-moving particle varies inversely with time. The following table gives the distribution.

Time (t)	1	2	4	5	8	10	12.5	20	25	50
Velocity (v)	100	50	25	20	12.5	10	8	5	4	2

- From the table, graph the variation.

Solution:



- Find the constant of variation.

Solution:

For inverse variation use $tv = k$, where k is the constant of variation.

From the table, $t = 4$ when $v = 25$.

$$tv = k$$
$$4(25) = k$$
$$100 = k$$
$$k = 100$$

The constant of variation is 100.

- Write the equation that describes the relation.

Solution:

The equation that describes the relation is $tv = k$. Since $k = 100$, $tv = 100$ or $v = \frac{100}{t}$.

You have found the equation of an inverse variation if a table of values or a graph is given. Now find an equation for an inverse variation if a descriptive situation is given.

Example 6

The temperature (T) of the air in degrees Celsius varies inversely with the height (h) in metres above sea level. When $h = 200$ m, $T = 8°C$. Find the constant of variation and the variation equation.

Solution:

If T varies inversely with h, write $T \propto \frac{1}{h}$. This implies that $hT = k$.

You are told that when $h = 200$, $T = 8$.

$$hT = k$$
$$200(8) = k$$
$$1600 = k$$
$$k = 1600$$

The constant of variation is 1600. Therefore, the equation of the inverse variation is $hT = 1600$ or $T = \frac{1600}{h}$.

Look at the next example.

When a relation states ***a*** **varies inversely as** ***b*** **varies,** think of $a \propto \frac{1}{b}$ and $ab = k$.

Example 7

The speed (s) of a sprinter running a 200 m race varies inversely with the amount of time (t) in seconds that it takes to finish the race.

- Using $s = 10$ m/s and $t = 20$ s, find s when $t = 12.5$ s.

Solution:

Note that s varies inversely as t varies; thus, $s \propto \frac{1}{t}$ or $st = k$.

Substitute 10 for s and 20 for t.

$$10(20) = k$$
$$200 = k$$
$$k = 200$$

The constant of variation is 200.

The equation for this inverse variation is $st = 200$ or $s = \frac{200}{t}$.

You were asked to find s when $t = 12.5$ s.

Substitute $t = 12.5$.

$$s = \frac{200}{t}$$
$$s = \frac{200}{12.5}$$
$$s = 16$$

When $t = 12.5$ s, the speed is 16 m/s.

Note that when the speed increases, the time decreases.

- Find the value of t if $s = 25$ m/s.

Solution:

Use the equation $st = 200$.

Substitute $s = 25$, and solve for t.

$$25t = 200$$
$$t = \frac{200}{25}$$
$$t = 8$$

Therefore, $t = 8$ s when s is 25 m/s.

Now that you have become familiar with inverse variation, try some of the following questions.

Do at least four of the following.

1. Which of the following situations represent an inverse variation?

   a. The faster you drive, the less time it takes to complete the journey.

   b. The more powerful the lawn mower, the less time it takes to cut the grass.

   c. The faster you drive, the greater the distance travelled.

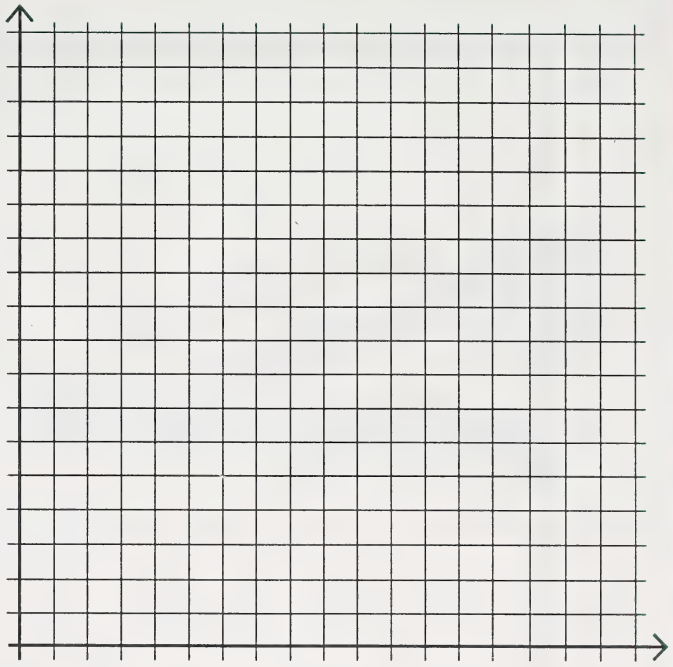
2. Which of the following is an example of inverse variation?

   a. $s = kt$ b. $s = \frac{t}{k}$

   c. $s = \frac{k}{t}$ d. $r = kv$

3. Calculate the value of the constant of variation for the inverse variation $t = \frac{k}{v}$. Graph the relation. Remember that $t = \frac{k}{v}$ can be written as $tv = k$.

t	6	8	12	16	24	40	48
v	80	60	40	30	20	12	10



4. Write the equation for the following if the constant of variation is k.

   a. The current (I) is inversely proportional to the resistance (R).

   b. The velocity (v) is inversely proportional to the time (t).

   c. The length (l) of a field is inversely proportional to the width (w).

5. The time to travel a distance is inversely proportional to the speed.

   a. Write an equation to represent this variation.

   b. When the speed is 80 km/h, the time required is 2 h. How much time is required when the speed is 100 km/h?

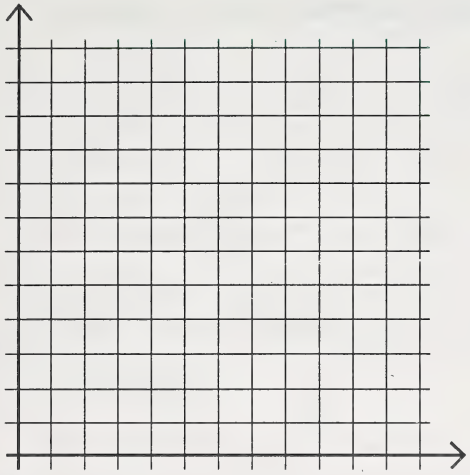
6. The constant of variation (k) is 48.

   a. Complete the table of values for the inverse relation $R = \frac{k}{I}$, where R is resistance and I is current.

   Remember that $R = \frac{k}{I}$ can be written as $RI = k$.

Resistance (R)	48	24	16	12	10	8	6	4.8	4	3	2	1
Current (I)												

b. Graph the relation.



For solutions to **Activity 1**, turn to the **Appendix, Topic 3**.

Activity 2

Graph and solve problems leading to partial variation.

Now that you have learned about direct variation and inverse variation, go on to study partial variation.

For example, when you rent a boat, part of the cost is fixed and part of it varies with time or with usage.

Such a variation involving both a fixed amount and a variable amount is defined as **partial variation**.

In the previous diagram the $25 is the **fixed cost** and the $3/h is the **variable cost**.

The total cost is partly constant and partly variable with the number of hours used.

The partial variation equation looks like the following:

$$C = F + kt$$

In the previous equation, C is the total cost, F is the fixed cost, and kt is the variable part of the cost. In this case, t is the time in hours and k is the constant of variation.

In order to solve a problem involving partial variation, you need to determine the constant part and the variable part. These two are added to find the total. Look at the following example.

Example 8

Rena rented a car for a day. The rental agency charged a flat rate of $30 per day, plus $0.05/km. Rena drove the car for 550 km before returning it. How much did she pay for renting the car that day?

Solution:

$C = F + kn$, where C = total cost,

F = the flat rate,

k = constant of variation, and

n = number of kilometres

$$C = F + kn$$
$$= 20 + 0.05(550)$$
$$= 20 + 27.50$$
$$= 47.50$$

Rena paid $47.50 to rent the car for the day.

What kind of graph will result from partial variation? Follow the next example.

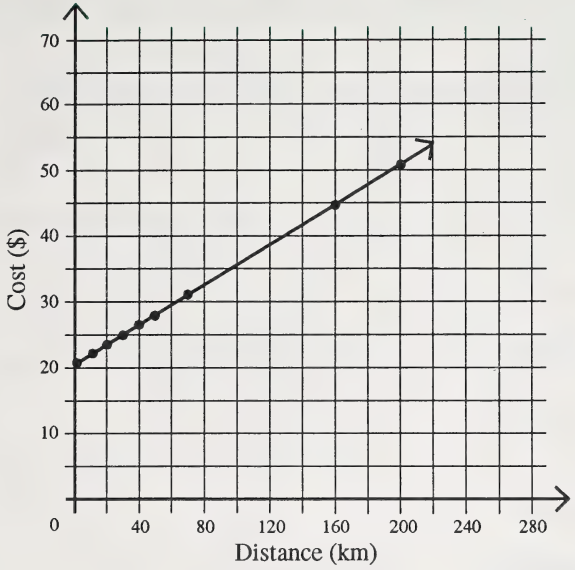
Example 9

Braheem rented a snowmobile from Gladue Snowmobiles. The terms were $21 per day plus $0.15/km. The following table shows a breakdown of the relation between cost and distance for one day's rental.

Use the following table to make a graph.

Distance (km)	0	10	20	30	40	50	70	160	200
Cost ($)	21.00	22.50	24.00	25.50	27.00	28.50	31.50	45.00	51.00

Solution:



Study the graph in Example 9, and note the following.

- The graph is a straight-line graph.
- The dependent variable or cost is on the vertical axis, and the independent variable or distance is on the horizontal axis.
- The line of the graph does not pass through the origin.
- Each pair of values from the table of values determines an ordered pair on the graph.
- The ratio $\frac{\text{cost}}{\text{distance}}$ is not constant. This ratio is found by using $\frac{\text{dependent variable}}{\text{independent variable}}$.

Some of the ratios would $\frac{21}{0}$ which is undefined, $\frac{22.50}{10}$ or 2.25, $\frac{24}{20}$ or 1.2, $\frac{25.50}{30}$ or 0.85, and so on.

In a direct variation, this ratio is a **constant**. Therefore, the previous relation is not a direct variation.

Consider the following examples that show how partial variation is used in everyday-life situations.

Example 10

Victoria Video handles the sales and servicing of video machines. For service calls, the company charges a $25 service charge plus a labour charge of $35/h. The following is a table of values for this relation.

Time (t) (h)	1	2	3	4	5
Cost (C) ($)	60	95	130	165	200

- Show that this situation represents a partial variation.

Solution:

This is a partial-variation problem because the charge consists of two parts: a fixed service charge and a labour charge which varies with the number of hours spent repairing the video machine.

- From the table, show that $\frac{C}{t}$ is not constant.

Solution:

If $\frac{C}{t}$ is constant, you have a direct variation. From the table, you can see the following.

For the first pair, $\frac{C}{t} = \frac{60}{1}$

$= 60$

For the second pair, $\frac{C}{t} = \frac{95}{2}$

$= 47.5$

For the third pair, $\frac{C}{t} = \frac{130}{3}$

$= 43.\dot{3}$

For the fourth pair, $\frac{C}{t} = \frac{165}{4}$

$= 41.25$

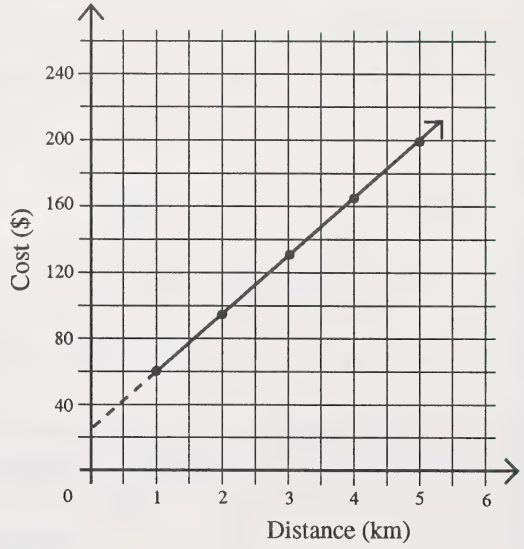
For the fifth pair, $\frac{C}{t} = \frac{200}{5}$

$= 40$

These calculations show that $\frac{C}{t}$ is not constant because it is not the same for all the ordered pairs.

- Use the data table to draw the graph.

Solution:



- Extrapolate to find the cost of zero hours.

Solution:

In order to find the cost at time zero, extrapolate from the graph. The cost when the time is zero is \$25. This is the fixed part of the cost.

In Example 10, the cost consists of two parts.

Cost = service charge + \$35 / h

$C = 25 + 35h$

The cost is \$25 plus \$35 times the number of hours.

This equation describes the partial-variation situation.

You may wish to use this equation to solve other problems. For example, if a video machine required six hours of repairs, what would the charge be for this service?

The charge would be calculated simply by using the formula $C = s + 35h$.

$C = 25 + 35h$

$= 25 + 35(6)$

$= 25 + 210$

$= \$235$

The charge would be \$235 for 6 h of work.

Example 11

The cost (C) for printing executive business cards is a fixed charge of \$80 for the first 1000 cards plus 5¢ for each additional card. The minimum order is 1000 cards.

- Write an equation that represents the relationship between the cost (C) and the number of cards printed (n) **over** the minimum order of 1000 cards.

Solution:

Cost = fixed amount + variable amount

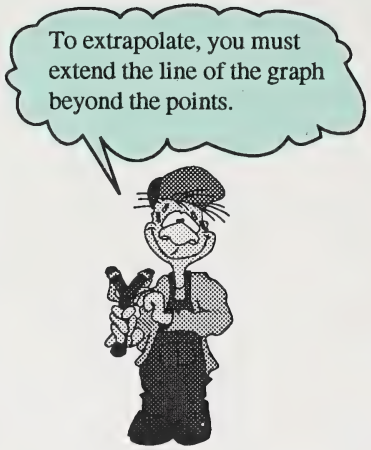
$C = \$80 + \$0.05n$

The equation is $C = 80 + 0.05n$.

BUSINESS CARDS

\$80 for 1000
plus
5¢ for each additional card

Get your cards now!



- What is the cost of printing 1900 cards?

Solution:

$$n = 1900 - 1000$$
$$= 900$$

$$C = 80 + 0.05n$$
$$= \$80 + \$0.05(900)$$
$$= \$80 + \$45$$
$$= \$125$$

The cost of printing 1900 cards is \$125.

- What is the cost of printing 4300 cards?

Solution:

$$n = 4300 - 1000$$
$$= 3300$$

$$C = 80 + 0.05n$$
$$= \$80 + \$0.05(3300)$$
$$= \$80 + \$165$$
$$= \$245$$

The cost of printing 4300 cards is \$245.

You have learned to identify a given partial variation and write its equation. These skills are needed when drawing graphs.

Example 12

The cost of operating a horseback-riding stable depends on a fixed cost of \$30/day plus \$5 for each riding team.

- Write an equation that represents the relation between the cost (C) and the number of riding teams (n).

Solution:

Cost = fixed amount + variable amount

$\therefore C = 30 + 5n$

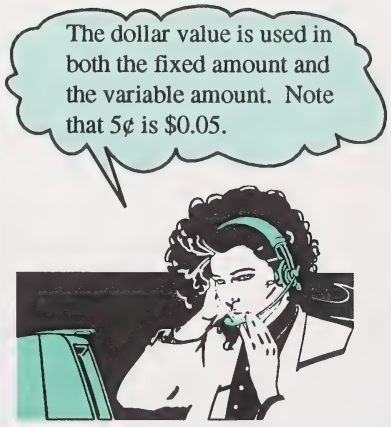
- What would the cost be for two riding teams, for four riding teams, and for nine riding teams?

Solution:

For two riding teams, $n = 2$.

$$C = 30 + 5n$$
$$= 30 + 5(2)$$
$$= 30 + 10$$
$$= 40$$

The cost would be \$40 for two riding teams.



For four riding teams, $n = 4$.

$$\begin{aligned} C &= 30 + 5n \\ &= 30 + 5(4) \\ &= 30 + 20 \\ &= 50 \end{aligned}$$

The cost would be \$50 for four riding teams.

For nine riding teams, $n = 9$.

$$\begin{aligned} C &= 30 + 5n \\ &= 30 + 5(9) \\ &= 30 + 45 \\ &= 75 \end{aligned}$$

The cost would be \$75 for nine riding teams.

- Construct a table of values from your calculations.

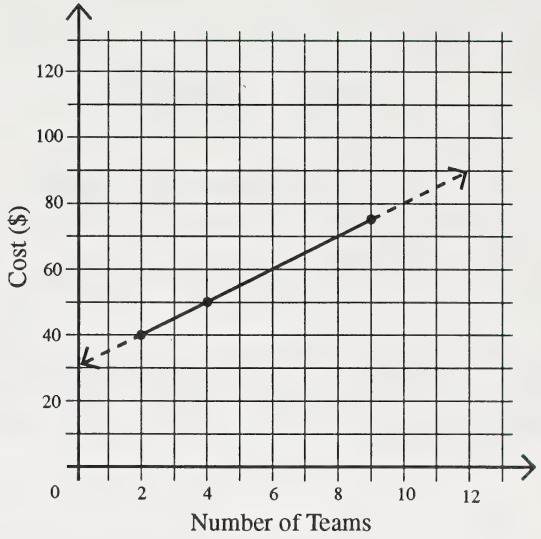
Solution:

To construct a table of values, use the equation $C = 30 + 5n$. Then assign values to n and find the corresponding C-values.

n	2	4	9
C	40	50	75

- Draw a graph for this relation.

Solution:



If you were to extend the graph downward, the line would meet the vertical axis at 30. Recall that 30 is the fixed amount.

Most partial variation problems involve two variables. You can be given the value of either of these variables and then asked to solve for the value of the remaining one.

Example 13

For each service call, Keith's Appliance Service & Repair Company charges $25 plus a labour rate of $28 per hour.

- A service bill totalled $145. How many hours of work does this represent?

Solution:

First derive the partial-variation equation.
Let C be the cost and t be the time.
If $C = 25 + 28t$, find t when $C = 145$.

$$\begin{aligned} C &= 25 + 28t \\ 145 &= 25 + 28t \\ 145 - 25 &= 28t \\ 120 &= 28t \\ 4.3 &= t \end{aligned}$$

The service person spent about 4.3 h on the repair job.

- What would be the charge for service if a repair took 5.5 h?

Solution:

Find C when $t = 5.5$.

$$\begin{aligned} C &= 25 + 28t \\ &= 25 + 28 \times 5.5 \\ &= 25 + 154 \\ &= 179 \end{aligned}$$

The total charge for the service would be $179.

Try the following questions.

Do any four of the following.

1. Redeana sells water-treatment dispensers for a company. She is paid a flat rate of $120/week plus 15% commission on sales during the week. Each water-treatment dispenser is sold for $350.

   a. Write an equation for Redeana's weekly income.

   b. Determine Redeana's total income for a week if she sells the following numbers of dispensers.

      i. ten dispensers in one week
      ii. sixteen dispensers in one week

2. If you rent a truck, it will cost a fixed \$18/d plus 3¢/km or \$0.03/km.

   a. How much will it cost if you rent a truck for a day and drive 450 km?

   b. How much will it cost if you rent a truck for two days and drive 750 km?

3. What is the fixed part and the variable part in each of the following?

   a. $H = 150 + 50h$ b. $T = 36 + 0.03t$

   c. $F = 3.45 + 1.5n$ d. $V = 2\pi + \frac{4r}{3}$

4. The cost of renting a sailboat is a flat fee of \$17 plus \$30 for each day. The equation for this partial variation is $C = 17 + 30n$.

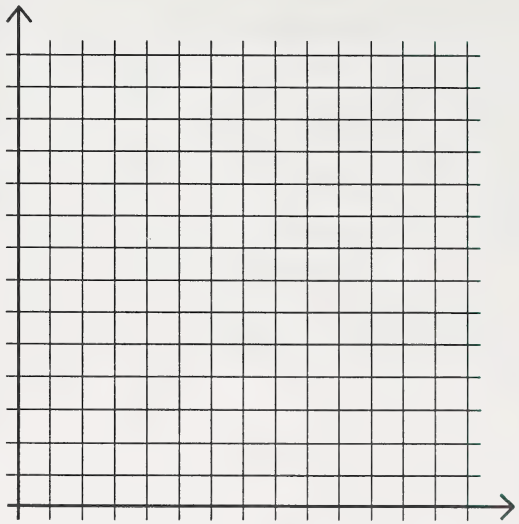
   a. What do C and n represent?

   b. What is the cost when $n = 2$ and when $n = 10$?

   c. For how long was the sailboat rented if it cost \$647? For how long was it rented if it cost \$107?

5. a. Use the following table to construct a graph.

Distance (km)	0	50	100	150	200
Cost (\$)	18.00	19.50	21.00	22.50	24.00



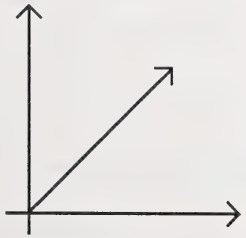
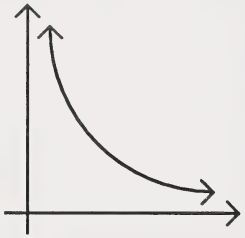
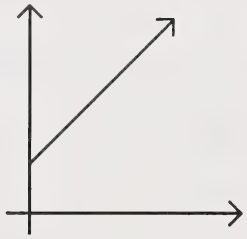
   b. What is the cost for travelling a distance of 250 km?

For solutions to **Activity 2**, turn to the **Appendix**, **Topic 3**.

Audio Activity

The following is an audio activity which will help you to draw together the major concepts in this unit. Listen to the audiotape before you proceed with the major assignment of this unit. Insert the audiotape entitled Mathematics 13, Unit 5 – Variation and follow the instructions on the tape.

Variation

	Direct Variation	Inverse Variation	Partial Variation
1	One variable varies directly as another variable varies.	One variable varies inversely as another variable varies.	One variable varies as a fixed amount plus another variable varies.
2	The graph of a direct variation is a straight line that always passes through the origin. 	The graph of an inverse variation does not pass through the origin and is not a straight line. 	The graph of a partial variation is a straight line, but it does not pass through the origin. 

	Direct Variation	Inverse Variation	Partial Variation
3	In general, $y = kx$ or y equals some constant times x.	In general, $y = \frac{k}{x}$ or $xy = k$. The product of two variables is a constant.	In general, $C = F + kT$ or C equals a fixed amount plus a variable amount.
4	Direct variation problems have a constant of variation.	Inverse variation problems have a constant of variation.	Partial variation problems do not have a constant of variation.
5	Examples of direct variation: a. The stretch of a spring varies directly as the force applied varies. b. The circumference of a circle varies directly as the radius varies. c. The amount of interest varies directly as the rate varies. d. Distance varies directly as time varies.	Examples of inverse variation: a. The frequency of a string varies inversely as its length varies. b. The current in a wire varies inversely as the resistance of the wire varies. c. The length of a rectangle varies inversely as the width for a given area varies.	Examples of partial variation: a. A salesperson in a clothing department earns \$300 per week plus 7% commission on the total sales for the week. b. A car rental company rents its cars at \$25/d plus \$0.15/km.
6	Translation of the variation examples from number 5 is as follows:		
	a. $S \propto F \quad S = kF$ b. c. d.	a. $F \propto \frac{1}{l} \quad F = \frac{k}{l}$ b. c.	a. $T = \$300 + 0.07S$ b.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.

Extra Help

When two quantities vary inversely, an increase in one quantity causes a decrease in the other quantity. Inverse variation also occurs when a decrease in one quantity creates an increase in the other quantity. Even though the change is opposite, the **product** of the two quantities remains equal. This value is known as the **constant of variation**.

In the following table, t represents the time in hours required to complete a journey at a given speed (s) in kilometres per hour.

s	20	30	40	60
t	12	8	6	4

As the speed increases, the time required to complete the journey decreases. You can say that t varies inversely as s varies or t is inversely proportional to s.

For any two ordered pairs (s_1, t_1) and (s_2, t_2), you can say that $s_1 t_1 = s_2 t_2$. This is always true for inverse variation.

Check this from the previous table.

$$\begin{aligned} s_1 t_1 &= s_2 t_2 \\ 20 \times 12 &= 30 \times 8 \\ 240 &= 240 \end{aligned}$$

Since this is always true, you can make the statement that $\frac{s_1}{s_2} = \frac{t_2}{t_1}$.

A situation in which the outcome is based on two parts is called a partial variation. One part is the fixed part, and the other is the variable part.

A general form of this variation is $y = b + mx$, where b is the fixed part and mx is the variable part.

Remember that $t \propto \frac{1}{s}$ is read as ***t* varies inversely as *s* varies**, or ***t* is inversely proportional to *s***.

Try the following examples.

Example 14

Suppose the area of a triangle is fixed, and the base and the height vary inversely. If a triangle has a base of 24 cm and a height of 16 cm, find the height of a triangle equal in area with a base of 18 cm.

Solution:

Since the base (b) is inversely proportional to the height (h), write $b \propto \frac{1}{h}$. This means that $\frac{b_1}{b_2} = \frac{h_2}{h_1}$ or $b_1 h_1 = b_2 h_2$, where $h_1 = 16$, $b_1 = 24$, and $b_2 = 18$.

Substitute the given values into $b_1 h_1 = b_2 h_2$ to find h_2.

$$\begin{aligned} b_1 h_1 &= b_2 h_2 \\ 24(16) &= 18h_2 \\ 18h_2 &= 24 \times 16 \\ 18h_2 &= 384 \\ \frac{18h_2}{18} &= \frac{384}{18} \\ h_2 &= 21.333\ldots \\ &= 21.\dot{3} \text{ or } 21\frac{1}{3} \end{aligned}$$

The height is $21\frac{1}{3}$ cm.

Example 15

The cost of producing a newsletter is partly constant and partly variable with the number printed. In order to print 50 copies, the total cost is \$53.60 and the variable cost is \$4.80. Find the cost of producing 2000 copies.

Solution:

First use the general form $y = b + mx$, where b is the fixed cost and mx is the variable cost. Substitute $y = \$53.60$ and $mx = \$4.80$ into $y = b + mx$ and solve for b.

$$\begin{aligned} y &= b + mx \\ 53.60 &= b + 4.80 \\ b &= 53.60 - 4.80 \\ b &= 48.80 \end{aligned}$$

Use $mx = \$4.80$ and $x = 50$ to solve for m which is the extra cost per copy produced.

$$\begin{aligned} mx &= 4.80 \\ m \times 50 &= 4.80 \\ \frac{m \times 50}{50} &= \frac{4.80}{50} \\ m &= 0.096 \end{aligned}$$

The extra cost of producing one newsletter is \$0.096.

To find the cost of producing 2000 copies, use $y = b + mx$, where $b = 48.80$, $m = 0.096$, and $x = 2000$.

$$\begin{aligned} y &= b + mx \\ &= 48.80 + (0.096)(2000) \\ &= 48.80 + 192 \\ &= 240.80 \end{aligned}$$

The total cost for printing 2000 copies is \$240.80.

Try the following questions.

1. Study the inverse-variation table, and then answer the questions.

C	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	2	4	8
t	8	4	2	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

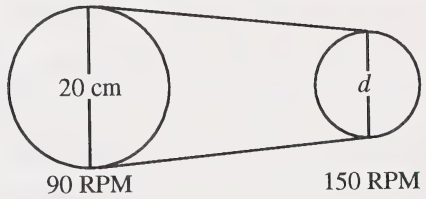
   a. What is the constant of variation?

   b. Write the equation for the relation.

   c. If $C = 16$, what is t?

   d. If $t = \frac{1}{3}$, what is C?

2. When a small pulley and a large pulley are connected, the smaller revolves more rapidly than the larger. The large pulley has a diameter of 20 cm and revolves at 90 RPM. Find the diameter of the small pulley if it revolves at 150 RPM. In this problem you must compare the RPM and the diameter of each pulley. Note that the abbreviation **RPM** stands for **revolutions per minute**.



3. The total cost (C) of placing an advertisement in a magazine partly depends on a fixed cost, and partly varies directly with the number of words in the advertisement. If the fixed cost is \$51.50 and the cost per word is \$0.035, find the total cost of placing an advertisement made up of 500 words.

4. The expenses of a basketball tournament are partly constant and partly variable with the number of players who attend. The fixed part is \$150 and the total charge is \$2150. If there are twelve players, what is the charge for each player who participates?

For solutions to **Extra Help**, turn to the **Appendix**, **Topic 3**.

Extensions

In addition to what you have learned in this unit, it should be mentioned that in a variation, one variable may be squared in a manner corresponding to the changes in the other variable.

For inverse variation, this may be expressed as follows:

$$d \propto \frac{1}{t^2} \text{ or } d \propto \frac{k}{t^2} \text{ or } dt^2 = k$$

If ordered pairs such as (d_1, t_1^2) and (d_2, t_2^2) are given, you can write the following:

$$d_1 t_1^2 = d_2 t_2^2$$

This is sometimes referred to as the **inverse-square law**.

Look at the following example.

Example 16

Loggers in British Columbia are required to replant the trees they harvest. For a given area, the loggers discovered that the number of trees planted varies inversely as the square of the distance between the trees varies. If 300 trees are required when the distance is 6 m apart, what is the number of trees required when the distance is 12 m apart.

Solution:

Let the distance between the trees be d and the number of trees be t.

The inverse-square law states that $t \propto \frac{1}{d^2}$. Use $t_1 d_1^2 = t_2 d_2^2$, where $t_1 = 300$, $d_1 = 6$, and $d_2 = 12$. Find t_2.

$$\begin{aligned} t_1 d_1^2 &= t_2 d_2^2 \\ 300 \times (6)^2 &= t_2 \times (12)^2 \\ 300 \times 36 &= t_2 \times 144 \\ t_2 \times 144 &= 10\,800 \\ \frac{t_2 \times 144}{144} &= \frac{10\,800}{144} \\ t_2 &= 75 \end{aligned}$$

The number of trees required is 75.

Try the following questions.

1. In certain situations, d varies inversely as t varies. When $t = 80$, $d = 60$.

   a. What is the constant of variation?

   b. What is the equation that reflects this inverse variation?

   c. Find d when $t = 32$.

2. The brightness (E) of a light is inversely proportional to the square of the distance (d) from the light source. Use the formula $E = \frac{I}{d^2}$. Find d in metres given $E = 8.0$ lumens and $I = 128$ candles. In symbol form this situation would be $E \propto \frac{1}{d^2}$ which becomes $E \propto \frac{k}{d^2}$ or $Ed^2 = k$. In this problem the constant of variation is I, so $E \propto \frac{k}{d^2}$ becomes $E = \frac{I}{d^2}$ and $Ed^2 = k$ becomes $Ed^2 = I$.

For solutions to **Extensions**, turn to the **Appendix**, **Topic 3**.

Unit Summary

What You Have Learned

In this unit you studied how to do the following:

- Sketch graphs that relate to physical phenomena, and interpret graphs that deal with real-life variables.
- Interpolate and extrapolate data from graphs and tables.
- Calculate the constant, graph equations, and solve problems related to direct variation.
- Graph and solve problems leading to inverse variation.
- Graph and solve problems leading to partial variation.

You are now ready to complete the **Unit Assignment**.

Appendix

Solutions

Review

Topic 1 Graphing Relations

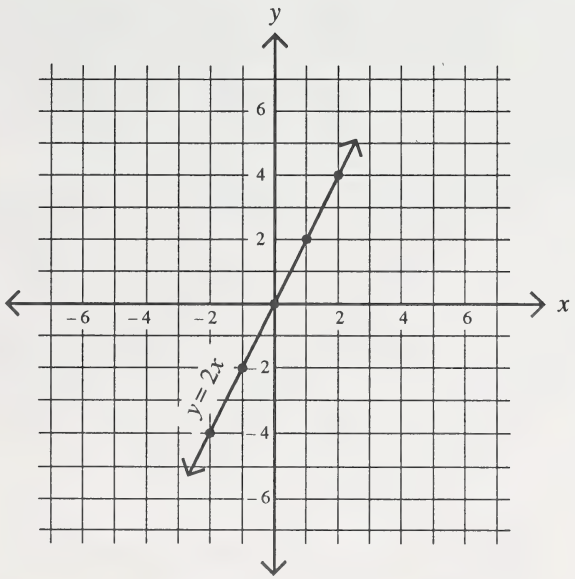
Topic 2 Direct Variations

Topic 3 Inverse and Partial Variations

Review

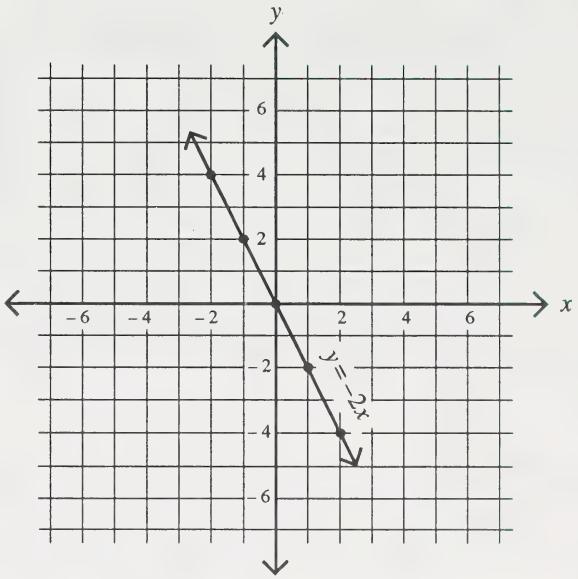
1. $y = 2x$

x	-2	-1	0	1	2
y	-4	-2	0	2	4



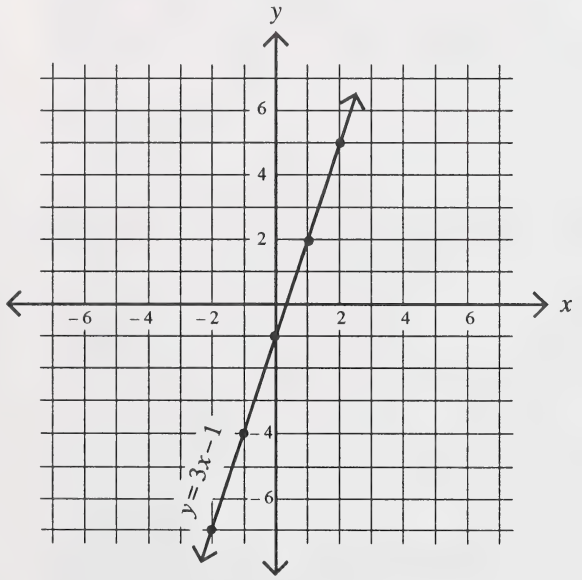
2. $y = -2x$

x	-2	-1	0	1	2
y	4	2	0	-2	-4



3. $y = 3x - 1$

x	-2	-1	0	1	2
y	-7	-4	-1	2	5

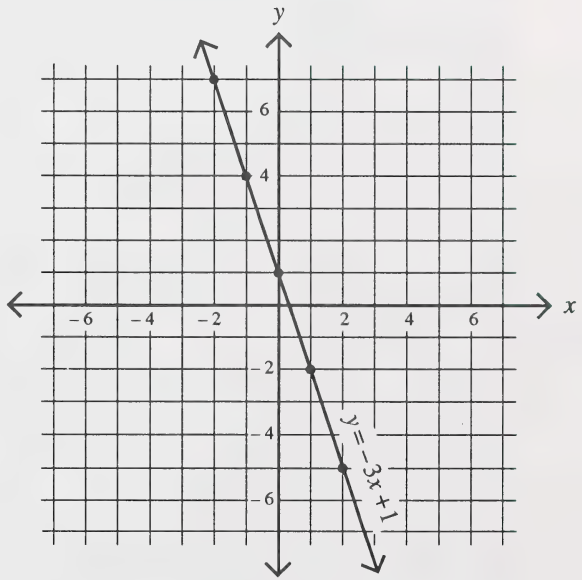


To find the first y-value, let $x = -2$ in $y = 3x - 1$.

$$\begin{aligned} y &= 3x - 1 \\ &= 3(-2) - 1 \\ &= -6 - 1 \\ &= -7 \end{aligned}$$

4. $y = -3x + 1$

x	-2	-1	0	1	2
y	7	4	1	-2	-5



To find the first y-value, let $x = -2$ in $y = -3x + 1$.

$$\begin{aligned} y &= -3x + 1 \\ &= -3(-2) + 1 \\ &= 6 + 1 \\ &= 7 \end{aligned}$$

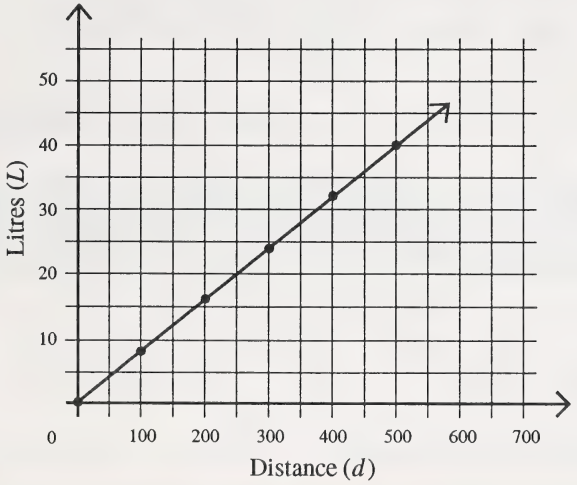
Exploring Topic 1

Activity 1

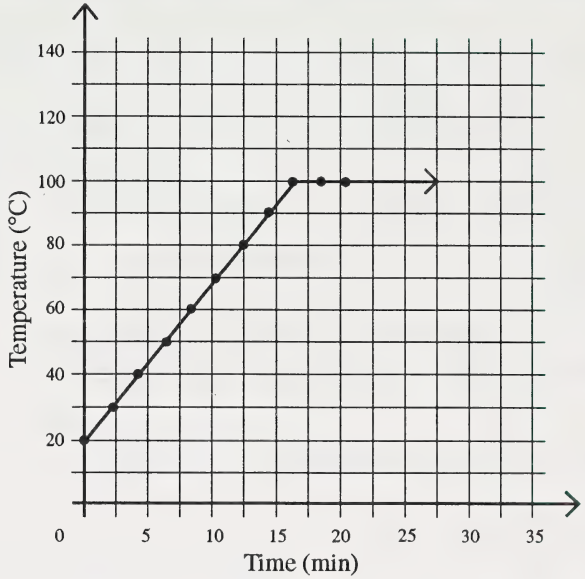
Sketch graphs that relate to physical phenomena, and interpret graphs that deal with real-life variables.

1.

d	0	100	200	300	400	500
L	0	8	16	24	32	40



2. Draw the graph using the table of values.



The water was the temperature of the room (20°C) when the experiment began. The temperature of the water rose uniformly with time. The temperature of the water reached 100°C in sixteen minutes. It remained at 100°C and did not exceed this temperature as more time went by.

3.

	CP	SP	FP	RT	BT	BP
°C	−40	−18	**0**	**20**	37	**100**
°F	−40	**0**	32	70	**100**	212

You will probably get a value of −2 °C for 32°F. However, the actual temperature for 32°F is 0°C. This inaccuracy was caused by the use of large scales along the vertical and horizontal axes.

4. a. Renata's energy curve might be high at 0800 because she has had a good night's rest. She probably wakes up in high spirits and is ready for the day.

   At 1400 she has finished lunch, had a rest, and is ready to continue working.

   At 2100 Renata has finished dinner, had a rest, and is excited about going out with her friends.

   b. Renata's energy level was at a really low point at about 1200 and again a bit before 1800. At midnight her energy level was again going down. These low energy levels were probably due to her need for food and/or rest.

5. a. Verica stopped to talk to her friend for ten minutes.

   b. The church is 14 km from Verica's home.

   c. If Verica had gone straight to church and had not stopped to chat with her friend, it would have taken $10 + 6 = 16$ min to go from her home to church.

6. a. In the expression $d = 5t$, t is the independent variable.

   b. In the expression $d = 5t$, d is the dependent variable.

   c. $d = 5t$ (Substitute 3 for t.)
   $d = 5(3)$
   $= 15$

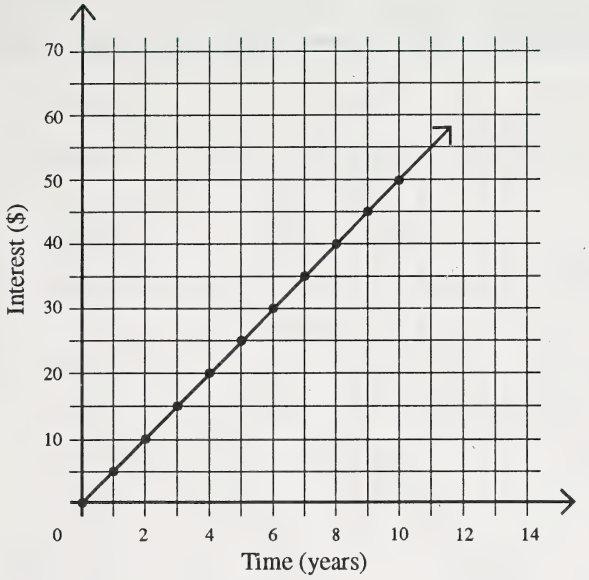
   The distance travelled in three minutes is 15 km.

7. a.

t	0	1	**2**	3	4	**5**	6	**7**	8	9
i	**0**	**5**	10	**15**	**20**	25	**30**	35	**40**	**45**

   b. In the equation $i = 5t$, the independent variable is t, and the dependent variable is i.

   c. The ordered pairs are (0, 0), (1, 5), (2, 10), (3, 15), (4, 20), (5, 25), (6, 30), (7, 35), (8, 40), and (9, 45).



b. i. In 3 h you would earn \$15.00.

ii. In 4 h you would earn \$20.00.

iii. In 11 h you would earn \$55.00.

iv. In 5.5. h you would earn $5.5 \times 5 = \$27.50$.

v. In 10.5 h you would earn $10.5 \times 5 = \$52.50$.

c. i. To earn \$35, it would take 7 h.

ii. To earn \$21, it would take $21 \div 5 = 4.2$ h.

iii. To earn \$30, it would take 6 h.

iv. To earn \$105, it would take $105 \div 5 = 21$ h.

v. To earn \$43.50, it would take $43.50 \div 5 = 8.7$ h.

Activity 2

Interpolate and extrapolate data from graphs and tables.

1. a.

Time	0	1	2	3	4	5	6	7	8	9	10	11	12
Earnings	0	**5**	**10**	15	**20**	25	**30**	**35**	40	**45**	**50**	55	**60**

2. Donna walks at a rate of 3 m/s.

a. In 21 s, she can walk $21 \times 3 = 63$ m.

b. $\frac{3\text{ m}}{1\text{ s}} = \frac{48\text{ m}}{x\text{ s}}$

$3x = 48 \times 1$

$3x = 48$

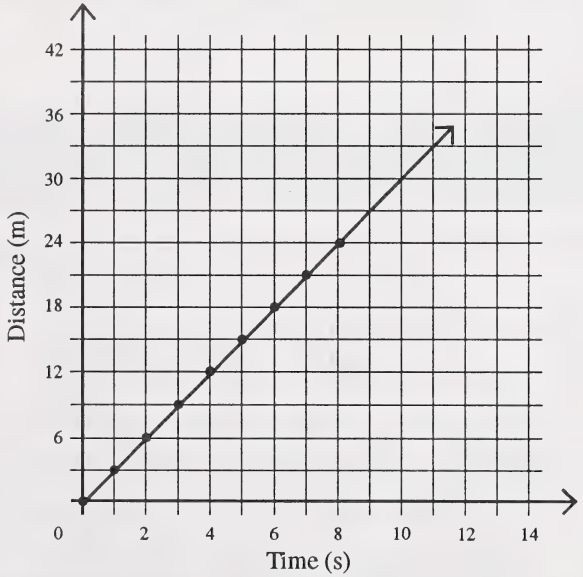
$\frac{3x}{3} = \frac{48}{3}$

$x = 16$

It will take Donna 16 s to walk 48 m.

c.

Time (s)	0	1	2	3	4	5	6	7	8
Distance (m)	0	3	6	9	12	15	18	21	24



Your answers may not be exactly the same in questions 3, 4, and 5.

3. a. (**5**, 200) b. (12, **480**)

   c. (21, **840**) d. (**4**, **160**)

   e. (**11**, 440) f. (**9**, 360)

4. a. In 20 days you would earn $800.

   b. In 17 days you would earn $680.

   c. In $24\frac{1}{2}$ days you would earn $980.

   d. In 19 days you would earn $760.

   e. In 27 days you would earn $1080.

5. a. To earn $600, it would take 15 days.

   b. To earn $540, it would take $13\frac{1}{2}$ days.

   c. To earn $100, it would take $2\frac{1}{2}$ days.

   d. To earn $1100, it would take $27\frac{1}{2}$ days.

   e. To earn $250, it would take $6\frac{1}{4}$ days.

6. a. i. The cost of printing 800 programs is $55.

      ii. The cost of printing 600 programs is $45.

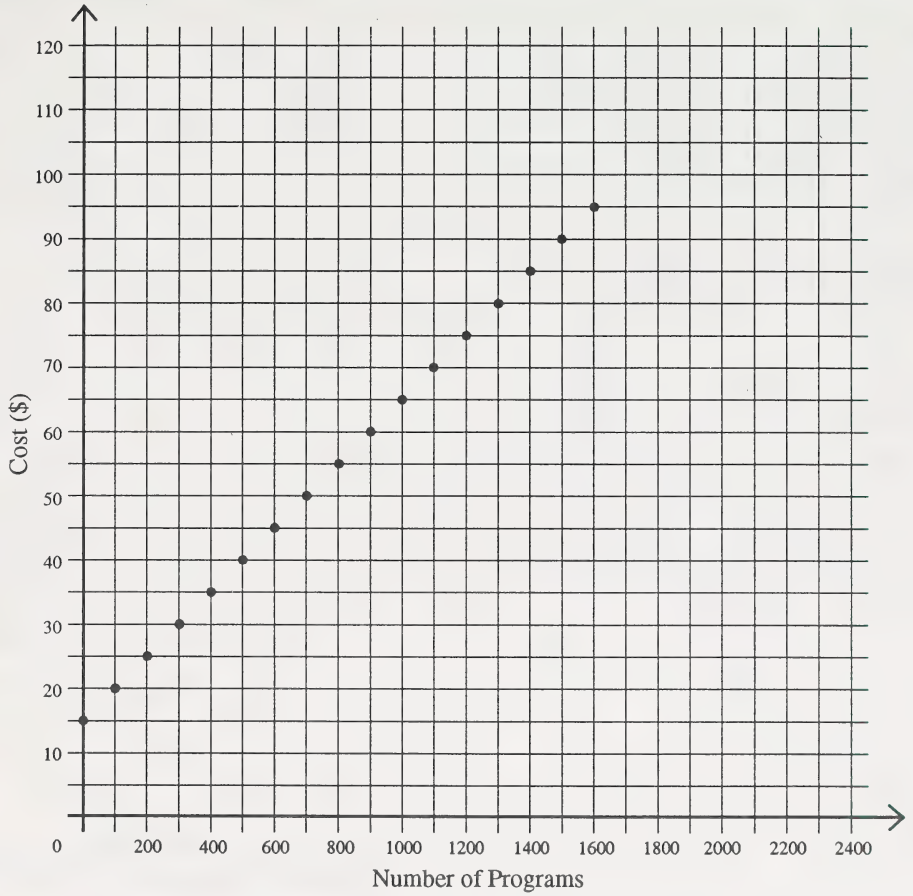
      iii. The cost of printing 1300 programs is $80.

   b. i. For $25 you can have 200 programs printed.

      ii. For $50 you can have 700 programs printed.

      iii. For $95 you can have 1600 programs printed.

The graph was extended so that the number of programs could be found by extrapolation.



c. It is necessary to have separate points and not a solid line on the graph because you cannot have part of a program. The components are not continuous.

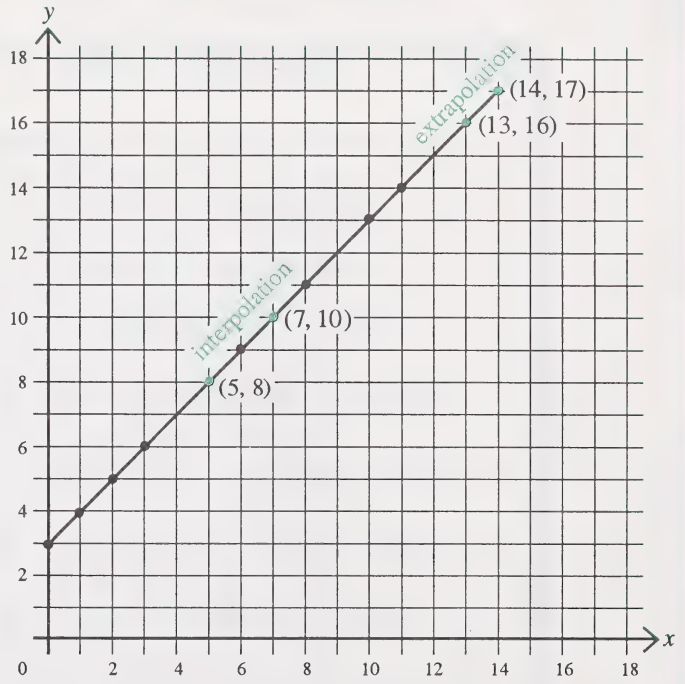
d. The point (0, 15) suggests that the minimum cost for programs is $15. You might consider the $15 as a start-up fee.

Extra Help

1. To find the bottom values, add 3 to the top values.

x	0	1	2	3	**6**	8	10	**11**
y	3	4	**5**	6	9	**11**	**13**	14

The ordered pairs are (0, 3), (1, 4), (2, 5), (3, 6), (6, 9), (8, 11), (10, 13), and (11, 14).

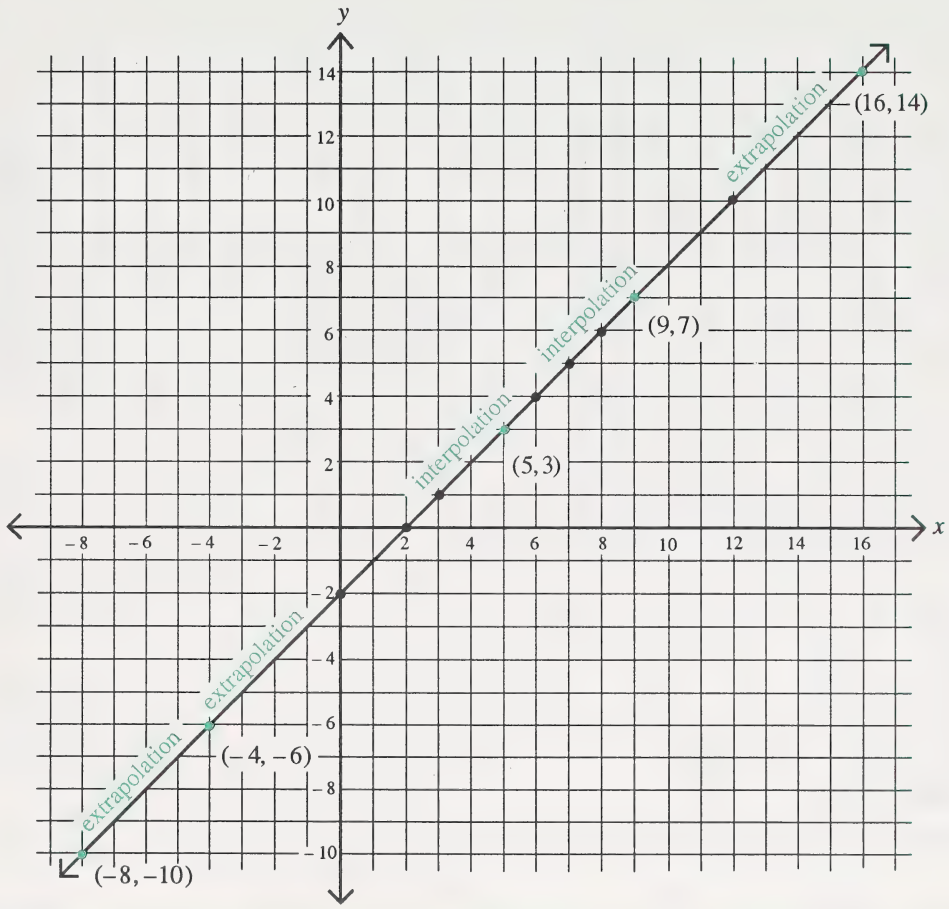


Two possible interpolated values are (5, 8) and (7, 10). Two possible extrapolated values are (13, 16) and (14, 17). There are others.

2. To find the bottom values, subtract 2 from the top values.

x	0	2	3	**6**	7	8	**10**	12
y	−2	0	1	4	**5**	**6**	8	**10**

The ordered pairs are (0, −2), (2, 0), (3, 1), (6, 4), (7, 5), (8, 6), (10, 8), and (12, 10).



Two possible interpolated values are (5, 3) and (9, 7). Three possible extrapolated values are $(-4, -6)$, $(-8, -10)$, and (16, 14). There are others.

Extensions

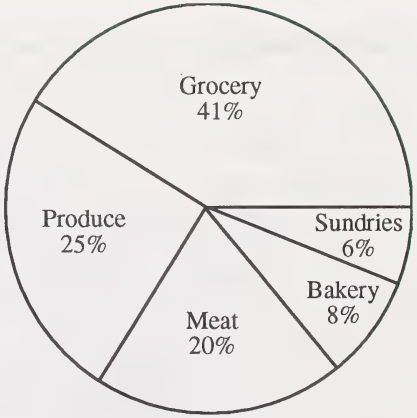
1. a. Grocery: $41\% = 0.41 \times 360°$
$= 147.6°$
$= 148°$

Produce: $25\% = 0.25 \times 360°$
$= 90°$

Meat: $20\% = 0.20 \times 360°$
$= 72°$

Bakery: $8\% = 0.08 \times 360°$
$= 28.8°$
$= 29°$

Sundries: $6\% = 0.06 \times 360°$
$= 21.6°$
$= 22°$



The total is 361° and not 360° because of the rounding which is needed to get whole numbers for 41%, 8%, and 6%.

b. Profit from sales for each department was as follows:

Grocery: 41% of \$3000 = \$1230

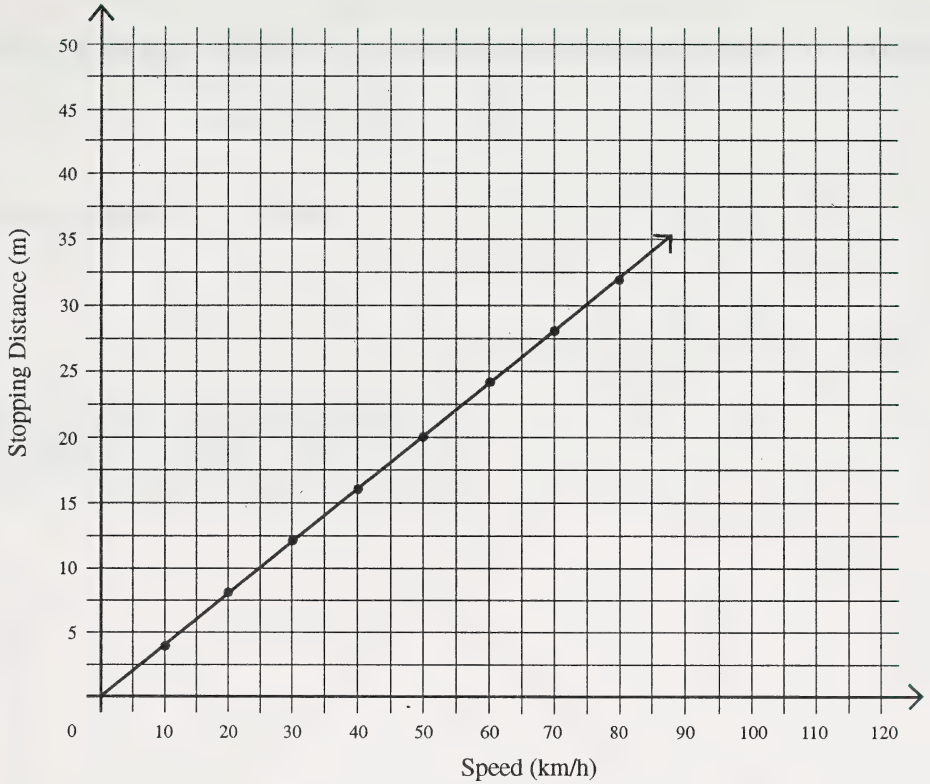
Produce: 25% of \$3000 = \$750

Meat: 20% of \$3000 = \$600

Bakery: 8% of \$3000 = \$240

Sundries: 6% of \$3000 = \$180

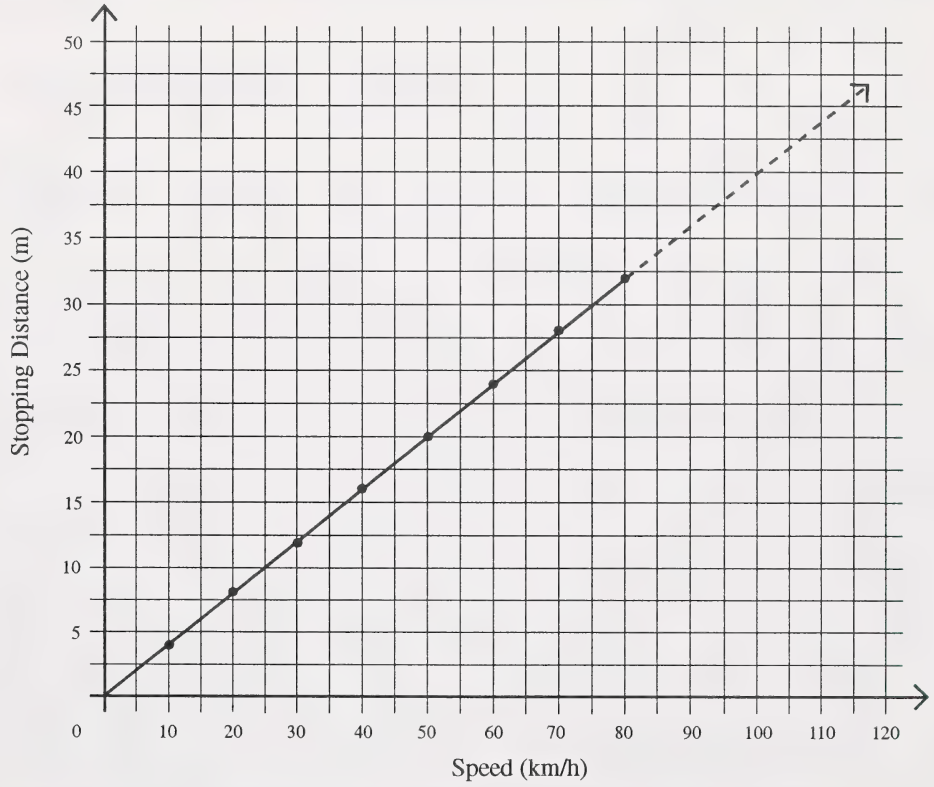
2. a.



b. According to the graph, if you were travelling 45 km/h, it would take you about 18.5 m to come to a stop.

c. According to the graph, if you were travelling 75 km/h, it would take you about 30 m to come to a complete stop after the brakes are applied.

d. If it took a vehicle 40 m to come to a stop, then the vehicle must have been travelling about 98 km/h.



Notice how the graph is extended to include a speed of 98 km/h and a distance of 40 m.

3.

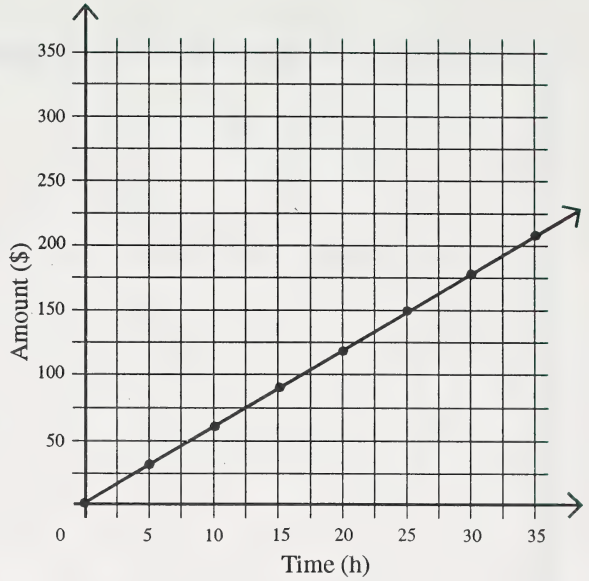
Time (h)	0	4.5	8	12	14	17	20	23
Distance (m)	0	3	5	8	9	11	13	15

Exploring Topic 2

Activity 1

Calculate the constant, graph equations, and solve problems related to direct variation.

1.



2. Direct variation is described in b, c, g, h, and i.

3. a. If $l \propto t$, then $l = k(t)$ is a direct relation. Thus, $k = \frac{l}{t}$.

When $l = 30$, $t = 6$.

$$k = \frac{l}{t}$$
$$= \frac{30}{6}$$
$$= 5$$

The following is done as a check.

When $l = 65,\ t = 13$.

$$k = \frac{l}{t}$$
$$= \frac{65}{13}$$
$$= 5$$

The constant of variation is 5.

b. If $l \propto w$, then $l = kw$ or $k = \frac{l}{w}$.

When $l = 4,\ w = 6$.

$$k = \frac{l}{w}$$
$$= \frac{4}{6}$$
$$= \frac{2}{3}$$

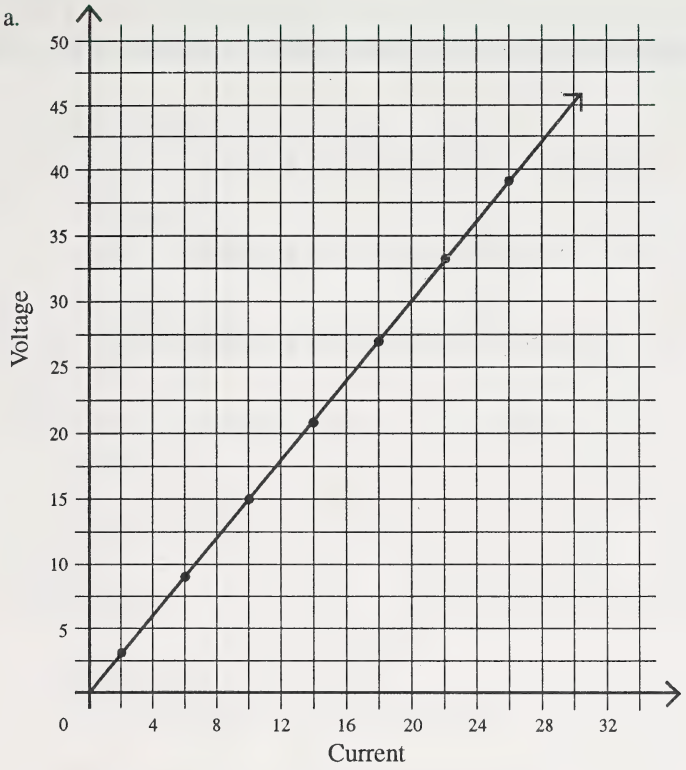
The following is done as a check.

When $l = 9,\ w = 13.5$.

$$k = \frac{l}{w}$$
$$= \frac{9}{13.5}$$ (Divide the numerator and the denominator by 3.)
$$= \frac{3}{4.5}$$ (Multiply the numerator and the denominator by 10.)
$$= \frac{30}{45}$$ (Divide the numerator and the denominator by 15.)
$$= \frac{2}{3}$$

The constant of variation is $\frac{2}{3}$.

4. a.



$V \propto I$

$V = kI$

The values for V and I can be taken from the table or the graph.

$3 = k(2)$ or $15 = k(10)$

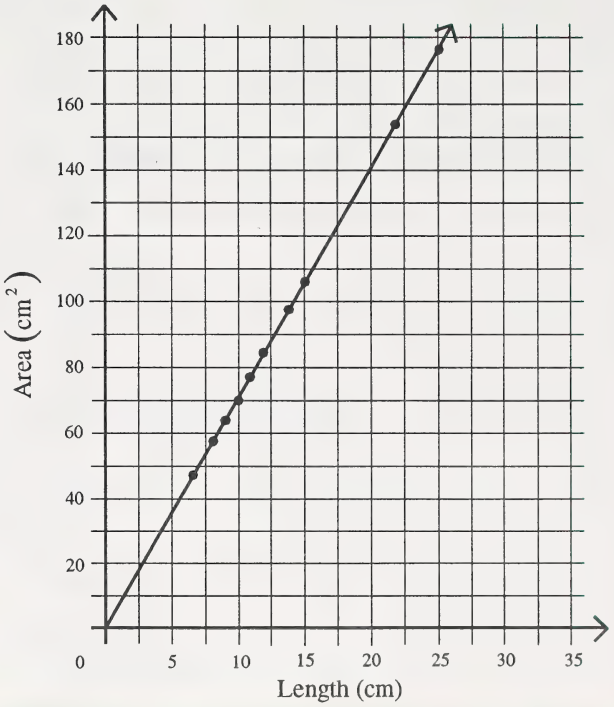
$2k = 3$ $\quad$ $10k = 15$

$\frac{2k}{2} = \frac{3}{2}$ $\quad$ $\frac{10k}{10} = \frac{15}{10}$

$k = \frac{3}{2}$ $\quad$ $k = \frac{3}{2}$

Therefore, $k = \frac{3}{2}$ and the equation is $V = \frac{3}{2}I$.

b.



$A \propto l$

$A = kl$

$63 = k(9)$ or $49 = k(7)$

$9k = 63$ $\quad$ $7k = 49$

$\frac{9k}{9} = \frac{63}{9}$ $\quad$ $\frac{7k}{7} = \frac{49}{7}$

$k = 7$ $\quad$ $k = 7$

Therefore, $k = 7$ and the equation is $A = 7l$.

5. a. For question 5 use the equation $V = \frac{3}{2}I$ obtained from the solution for 4a.

$$V = \frac{3}{2}I$$
$$= \frac{3}{2} \times 70$$
$$= \frac{210}{2}$$
$$= 105$$

i. When the current (I) is 70 amperes, the voltage (V) is 105 volts.

ii. When the current (I) is 10 amperes, the voltage (V) is 15 volts.

iii. When the current (I) is 105 amperes, the voltage (V) is 157.5 volts.

b. $V = \frac{3}{2}I$

$150 = \frac{3}{2}I$

$I = 150 \div \frac{3}{2}$

$= \frac{150}{1} \times \frac{2}{3}$

$= \frac{300}{3}$

$= 100$

i. When the voltage (V) is 150 volts, the current (I) is 100 amperes.

ii. When the voltage (V) is 63 volts, the current (I) is 42 amperes.

iii. When the voltage (V) is 20 volts, the current (I) is $13.\dot{3}$ amperes.

6. a. For question 6 use the equation $A = 7l$ from the solution for 4b.

$$A = 7l$$
$$= 7 \times 14$$
$$= 98$$

i. If the length (l) is 14 units, the area (A) is 98 square units.

ii. If the length (l) is 16 units, the area (A) is 112 square units.

iii. If the length (l) is 30 units, the area (A) is 210 square units.

b.
$$A = 7l$$
$$175 = 7l$$
$$7l = 175$$
$$\frac{7l}{7} = \frac{175}{7}$$
$$l = 25$$

i. If the area (A) is 175 square units, the length (l) is 25 units.

ii. If the area (A) is 217 square units, the length (l) is 31 units.

iii. If the area (A) is 42 square units, the length (l) is 6 units.

7. a. Note that $w \propto l$, where w is the weight and l is the length of extension.

Thus, $w = kl$, where k is the constant of variation.
When $w = 5$, $l = 2.5$.

$$w = kl$$
$$5 = k(2.5)$$
$$\frac{2.5k}{2.5} = \frac{5}{2.5}$$
$$k = 2$$

The equation of the variation is $w = 2l$.

b. When a weight of 30 kg is applied, the length is calculated as follows:

$$w = 2l$$
$$30 = 2l$$
$$\frac{2l}{2} = \frac{30}{2}$$
$$l = 15$$

The length of extension is 15 cm.

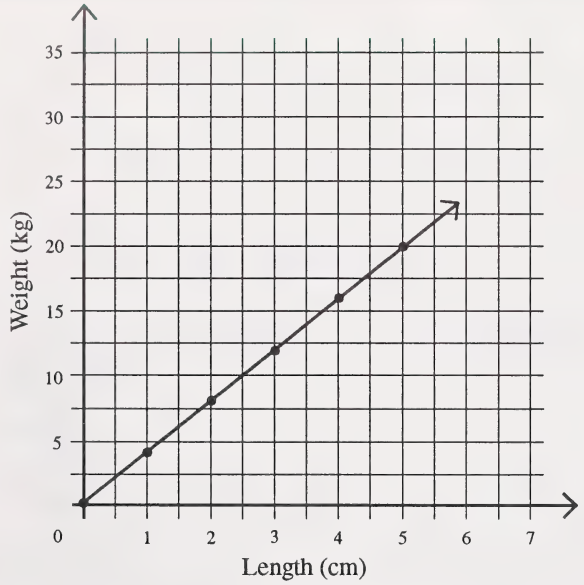
c. When the extension is 56 cm, the weight is calculated as follows:

$$w = 2l$$
$$= 2(56)$$
$$= 112$$

The weight is 112 kg.

Extra Help

1. a.



b. $w \propto l$

$w = kl$

From the table, $w = 12$ when $l = 3$.

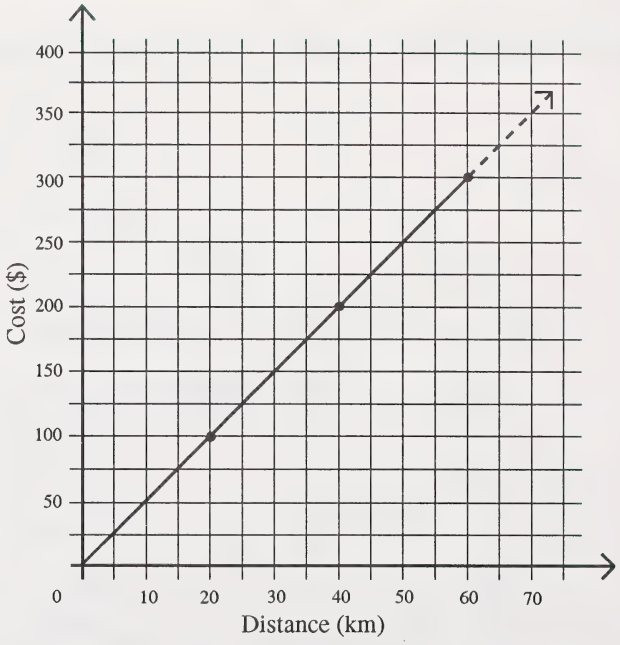
$12 = k(3)$

$k = \frac{12}{3}$

$= 4$

The constant of variation is 4.

2. a.



Your answers may not be identical to the following, but they should be close.

i. The cost for a trip of 20 km is $100.

ii. The cost for a trip of 40 km is $200.

iii. The cost for a trip of 44 km is $220.

iv. The cost for a trip of 60 km is $300.

v. The cost for a trip of 70 km is $350.

b. i. For $160 you can have a 32 km boat ride.

ii. For $180 you can have a 36 km boat ride.

iii. For $240 you can have a 48 km boat ride.

iv. For $280 you can have a 56 km boat ride.

v. For $350 you can have a 70 km boat ride.

Extensions

Your answers for questions 1 and 2 may not be exactly as given, but they should be close.

1. a. After 2 h, the bus travelled 60 km.

b. After 3.2 h, the bus travelled 100 km.

c. After 6.2 h, the bus travelled 200 km.

d. After 8 h, the bus travelled 260 km.

e. After 8.8 h, the bus travelled 280 km.

2. a. In order to travel 80 km, it would take 2.6 h.

b. In order to travel 120 km, it would take 3.8 h.

c. In order to travel 180 km, it would take 5.6 h.

d. In order to travel 240 km, it would take 7.4 h.

e. In order to travel 300 km, it would take 9.2 h.

3. If the bus travels 300 km in 9.2 h, the average speed is as follows:

$$s = \frac{300}{9.2}$$
$$= 32.608\ldots$$
$$\doteq 33$$

The average speed is about 33 km/h.

Notice that in question 1a the speed was $60 \div 2 = 30$ km/h. The two speeds are not the same. This is because the distances and times are estimated from the graphs and not calculated.

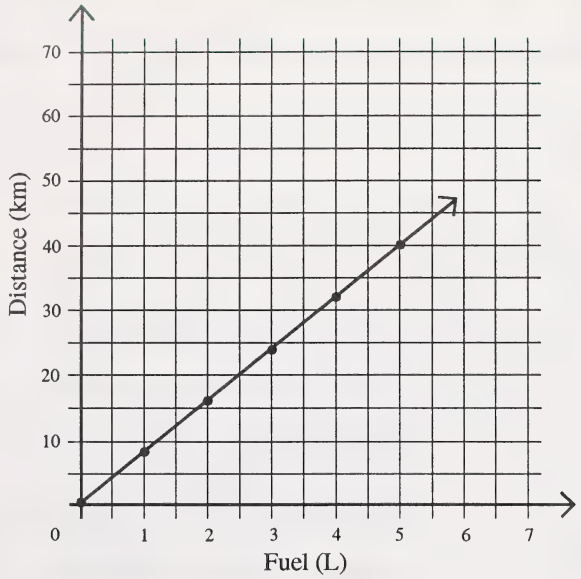
4. For this question you could say that d varies directly as L.

$d \propto L$ or $d = kL$

Here the dependent variable is distance and the independent variable is litres.

If $d = 8L$, then the table is as follows:

Distance (km)	8	16	24	32	40
Litres (L)	1	2	3	4	5



Exploring Topic 3

Activity 1

Graph and solve problems leading to inverse variation.

1. Both a and b represent inverse variation.
   In a, speed increases while time decreases. In b, the time required to cut the lawn decreases as the power of the lawn mower increases.

2. An example of inverse variation is $s = \frac{k}{t}$ (part c).
   As t increases, s decreases.

3. Choose two values of t and v from the table of values.
   Use $t = \frac{k}{v}$, where $t = 16$ when $v = 30$.

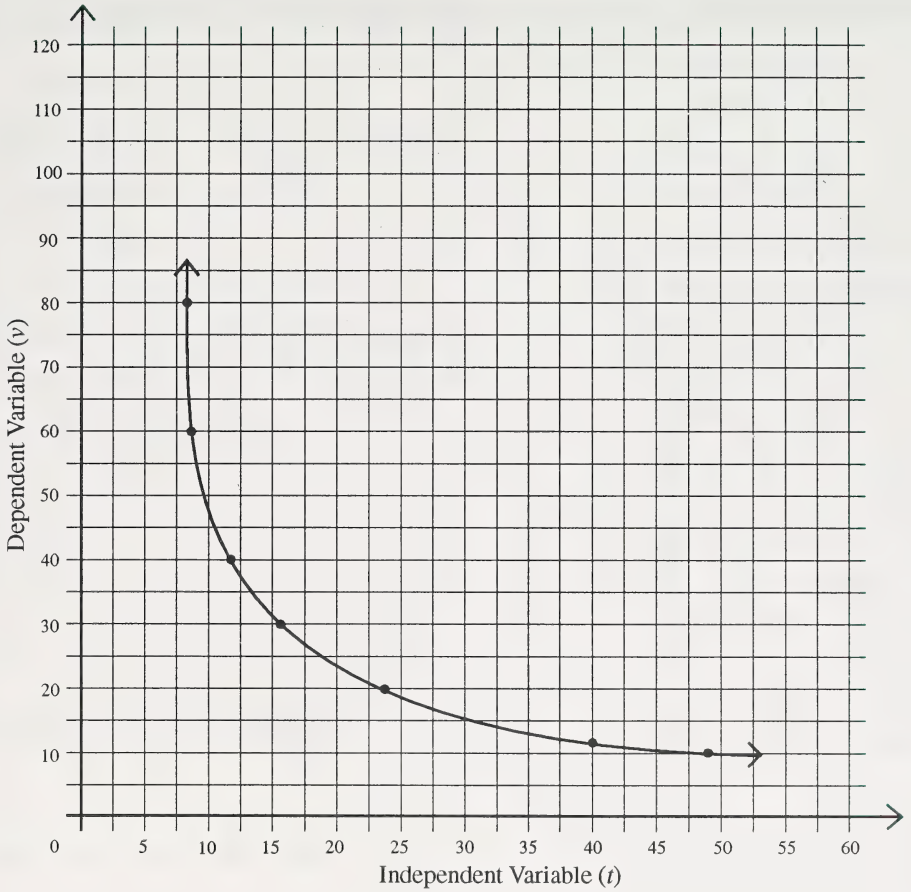
   $\frac{k}{v} = t$

   $k = tv$

   $= (16)(30)$

   $= 480$

   The constant of variation is $k = 480$.


120
110
100
90
80
70
60
50
40
30
20
10
0
5
10
15
20
25
30
35
40
45
50
55
60
Dependent Variable (v)
Independent Variable (t)

4. a. $I \propto \frac{1}{R}$ or $I = \frac{k}{R}$

Thus, $IR = k$, where k is the constant of variation.

b. $v \propto \frac{1}{t}$ or $v = \frac{k}{t}$

Thus, $vt = k$, where k is the constant of variation.

c. $l \propto \frac{1}{w}$ or $l = \frac{k}{w}$

Thus, $lw = k$, where k is the constant of variation.

5. a. $t \propto \frac{1}{s}$ or $t = \frac{k}{s}$

Thus, $st = k$, where k is the constant of variation.

b. When $s = 80$, $t = 2$.

$k = ts$

$= 2(80)$

$= 160$

Therefore, when $s = 100$,

$t = \frac{k}{s}$ or $k = ts$

$= \frac{160}{100}$ $160 = t \times 100$

$= 1.6$ h $t \times 100 = 160$

$\frac{t \times 100}{100} = \frac{160}{100}$

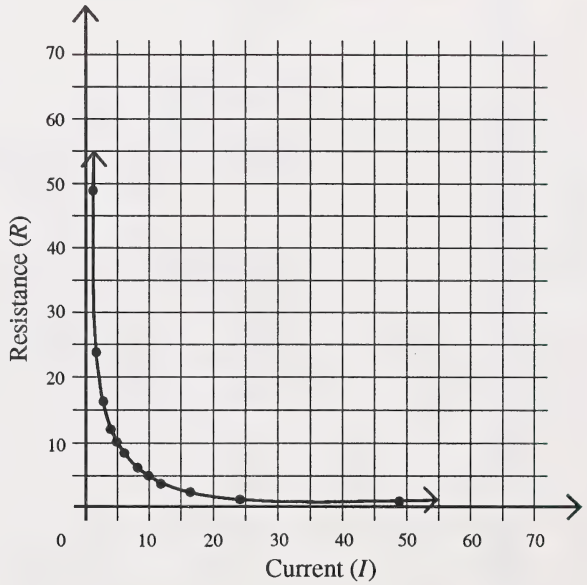
$t = 1.6$ h

6. a. $RI = k$

$RI = 48$

Resistance (R)	48	24	16	12	10	8	6	4.8	4	3	2	1
Current (I)	1	2	3	4	4.8	6	8	10	12	16	24	48

b.



Activity 2

Graph and solve problems leading to partial variation.

1. a. The formula for Redeana's weekly income is $I = 120 + 0.15s$.

   Note that I is the income, and s is the total sales.

   b. i. If Redeana sells ten water-treatment dispensers at \$350 each, the total amount of sales is $s = 10 \times \$350 = \$3500$.

   From the formula, you get the following:

   $I = 120 + 0.15s$

   $= 120 + 0.15(3500)$

   $= 120 + 525$

   $= 645$

   Redeana's total income that week would be \$625.

   ii. If Redeana sells sixteen dispensers at \$350 each, the total of her sales is $s = 16 \times \$350 = \$5600$.

   From the formula, you get the following:

   $I = 120 + 0.15s$

   $= 120 + 0.15(5600)$

   $= 120 + 840$

   $= 960$

   Redeana's total income that week would be \$960.

2. a. Let the cost be C and the number of kilometres be n.

   $\therefore C = 18 + 0.03n$ (where $n = 450$ km)

   $= 18 + 0.03(450)$

   $= 18 + 13.50$

   $= 31.50$

   The cost of renting a truck for the day is \$31.50.

   b. For two days the flat rate is $2 \times 18 = 36$.

   $\therefore C = 36 + 0.03n$ (where $n = 750$ km)

   $= 36 + 0.03(750)$

   $= 36 + 22.50$

   $= 58.50$

   The cost of renting a truck for two days is \$58.50.

3. a. $H = 150 + 50h$
   The fixed part is 150.
   The variable part is $50h$.

   b. $T = 36 + 0.03t$
   The fixed part is 36.
   The variable part is $0.03t$.

c. $F = 3.45 + 1.5n$

The fixed part is 3.45.

The variable part is $1.5n$.

d. $V = 2\pi + \frac{4r}{3}$

The fixed part is 2π.

The variable part is $\frac{4r}{3}$.

4. a. In $C = 17 + 30n$, C stands for total cost, and n stands for the number of days the sailboat was rented.

b. When $n = 2$, $C = 17 + 30n$

$= 17 + 30(2)$

$= 17 + 60$

$= 77$

The cost to rent the sailboat for two days is \$77.

When $n = 10$, $C = 17 + 30n$

$= 17 + 30(10)$

$= 17 + 300$

$= 317$

The cost to rent the sailboat for ten days is \$317.

c. If the cost of renting the boat is \$647, it was rented for the following number of days.

$C = 17 + 30n$

$647 = 17 + 30n$

$30n = 647 - 17$

$30n = 630$

$n = \frac{630}{30}$

$= 21$

The sailboat was rented for 21 days.

If the cost is \$107, the boat was rented for the following number of days.

$C = 17 + 30n$

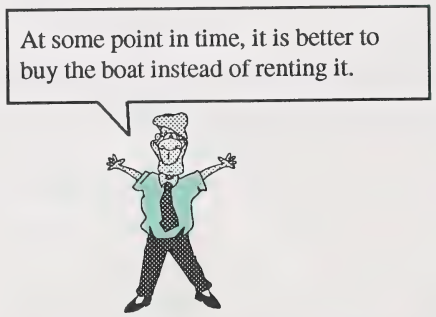
$107 - 17 = 30n$

$90 = 30n$

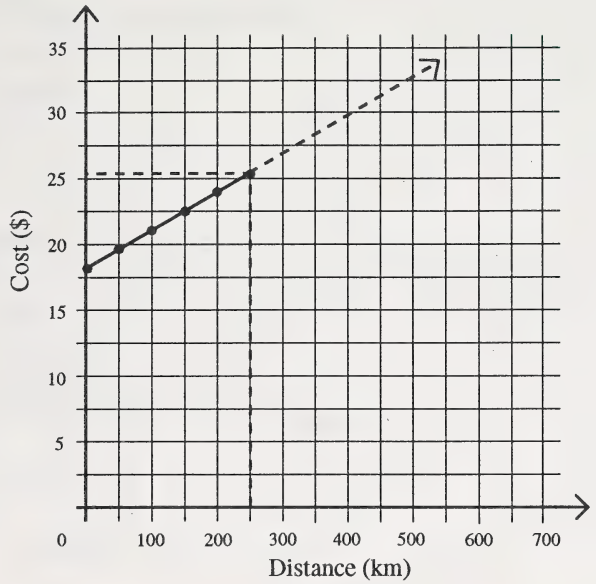
$3 = n$

$n = 3$

The sailboat was rented for 3 days.



5. a. The ordered pairs are (0, 18.00), (50, 19.50), (100, 21.00), (150, 22.50), and (200, 24.00).



b. From the graph, the cost of travelling a distance of 250 km is about $25.50.

Extra Help

1. a. Choose any value of C and the corresponding value of t.

$Ct = k$ or $Ct = k$

$\frac{1}{4} \times 4 = k$ $\quad 8 \times \frac{1}{8} = k$

$k = 1$ $\quad k = 1$

The constant of variation is 1.

b. For inverse variation, $C \propto \frac{1}{t}$ or $C = \frac{k}{t}$.

The equation is $C = \frac{1}{t}$ or $Ct = 1$.

c. Given that $C = 16$, find t.

$C = \frac{1}{t}$ or $Ct = 1$

$16 \times t = 1$

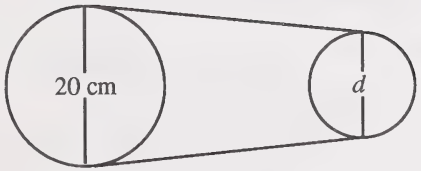
$16 = \frac{1}{t}$ $\quad \frac{16 \times t}{16} = \frac{1}{16}$

$t = \frac{1}{16}$ $\quad t = \frac{1}{16}$

d. Given that $t = \frac{1}{3}$, find C.

$$C = \frac{1}{t} \quad \text{or} \quad Ct = 1$$
$$= \frac{1}{\frac{1}{3}} \qquad C\left(\frac{1}{3}\right) = 1$$
$$= 1 \div \frac{1}{3} \qquad \frac{C \times \frac{1}{3}}{\frac{1}{3}} = \frac{1}{\frac{1}{3}}$$
$$= 1 \times \frac{3}{1} \qquad = 1 \times \frac{3}{1}$$
$$= 3 \qquad = 3$$

2.



$d \propto \frac{1}{s}$

The diameter of the large pulley is 20 cm and it rotates at 90 RPM.

$\therefore d = \frac{k}{s}$

$k = ds$

$= 20 \times 90$

$= 1800$

The constant of variation is 1800.

Use $k = ds$ and substitute 1800 for k and 150 for the speed of the small pulley.

$k = d(s)$

$1800 = d(150)$

$d = \frac{1800}{150}$

$= 12$

The diameter of the small pulley is 12 cm.

3. Use $y = b + mx$, where $b = \$51.50$, $m = 0.035$, and $x = 500$.

$y = 51.50 + (0.035)500$

$= 51.50 + 17.50$

$= 69.00$

The cost to place an advertisement of 500 words is \$69.

4. Use $y = b + mx$ where $y = \$2150$, $b = \$150$, and $x = 12$.

$2150 = 150 + 12m$

$2000 = 12m$

$\frac{12m}{12} = \frac{2000}{12}$

$m = 166.666\ldots$ or $166.\dot{6}$

$m \doteq 166.67$ (to the nearest cent)

The charge for each basketball player is about \$166.67.

Extensions

1. a. $d \propto \frac{1}{t}$

$d = \frac{k}{t}$

$k = dt$

$= 60(80)$

$= 4800$

The constant of variation is 4800.

b. The equation can be of the form $d = \frac{k}{t}$, $dt = k$, $d = \frac{4800}{t}$, or $dt = 4800$.

c. To find d when $t = 32$, use either $d = \frac{k}{t}$ or $dt = k$.

$d = \frac{k}{t}$ or $dt = k$

$= \frac{4800}{32}$ $d \times 32 = 4800$

$= 150$ $\frac{d \times 32}{32} = \frac{4800}{32}$

$d = 150$

2. To find d, use $E \propto \frac{I}{d^2}$ or $E = \frac{I}{d^2}$, where I is the constant of variation. If $E = 8.0$ lumens and $I = 128$ candles, substitute these values into $E = \frac{I}{d^2}$ to find d.

$E = \frac{I}{d^2}$

$8.0 = \frac{128}{d^2}$

$8.0d^2 = 128$

$\frac{8.0d^2}{8.0} = \frac{128}{8.0}$

$d^2 = 16$

$\sqrt{d^2} = \sqrt{16}$

$d = 4$

The distance from the light source is 4 m.

Mathematics 13

9MA13P35

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